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ABSTRACT

This mathematics support guide is intended for use by vocational students and instructors as a review of essential mathematics concepts and for problem-solving exercises in the vocations. It is designed to accompany the "Mathematical Skills Inventory," which tests mathematics skills, attitudes, and background. A section entitled Arithmetic Skills contains Review Notes and Problem Solving exercises in these areas: whole numbers, fractions, decimals, ratio, percents, and signed numbers. The Review Notes outline the fundamental steps in solving mathematics problems. These are presented as algorithms--a series of steps leading to the correct solution in a somewhat automatic way. The Problem Solving exercises give practice in each skill described. Problems are provided in these program areas: medical laboratory technician, mining technology, nursing, secretarial administration, business management, and computer technology. Three reference sections present introductions for a broad range of topics that are becoming more important in the vocational fields. They give students some of the vocabulary and concepts of modern mathematics and information processing. The sections are algebra (algebraic expressions, exponents in algebra, proportions in algebra, number sentences), concepts and problem solving (measurement, number forms, number theory, set theory, and graphs), and applications (metrics, calculators, computers, plane geometry, solid geometry). (YLB)

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STUDENT MATH SKILLS REFERENCE MANUAL

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1986

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Student Math Skills Reference Manual

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INTRODUCTION TO USE OF THE MANUAL

GOALS

This manual does not pretend to provide an in-depth coverage of the topics listed. It cannot replace the developmental mathematics instructors. However, we believe that it can serve as a valuable mathematics support guide.

Vocational students and instructors will best be able to use this manual as a review of essential mathematics concepts and for problem solving exercises in the vocations. The Review Notes outline the fundamental steps in solving math problems. These are presented as algorithms—a series of steps which will lead to the correct solution in a somewhat automatic way. Sometimes the student will discover a shorter way to find the answer. Other times, the manual will not cover a specific math principle. We hoped to produce a brief reference manual which will serve as a starting point for some students and as a refresher for others. The Problem Solving Exercises are arranged by the categories, giving practice in each of the skills described.

Finally, the Reference Sections provide introductions for a broad range of topics which are becoming more and more important in the vocational fields. Reading through these sections will give the student some of the vocabulary and concepts of modern mathematics and information processing. As a group, the Reference Sections allow the student to develop interests and gain breadth in mathematics and related concepts.

TEST SCORES

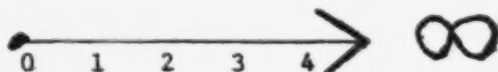
The manual is designed to accompany the Mathematical Skills Inventory (MSI). This evaluation tool tests math skills, attitudes, and background. Computer scoring of the MSI reprints scores in each of twenty categories which correspond to the twenty chapters in this manual. Thus, the MSI is a diagnostic test and not a test of ability. Scores show the profile of strengths and limitations in a student's math knowledge. The MSI will be particularly helpful to students who have not used mathematics for several years. If your score falls below 75% correct on any of the sub-areas, you should turn to those sections of this manual to examine the types of items which are giving you trouble. Study the Review Notes and then work the Problem Solving exercises for these areas. If more review is needed, it can be gained accurately by showing the exercise that you need help on.

1.1 WHOLE NUMBERS

REVIEW NOTES

The set of whole numbers includes those elements having the concept of unit positiveness or nothingness. In other words, the whole numbers are (0, 1, 2, 3, ...). There is no end to the concept, and it goes forever toward positive infinity.
(∞)

To work with these concepts we select a starting point (the origin) and assign its place as zero (0). To the right of zero are our unit concepts 1, 2, 3, and so on.

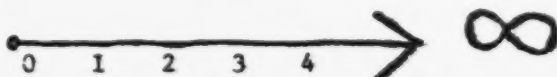


- 1.1.1 Thus counting, becomes the accumulation of units from the origin to a place on the number line shown above. Addition is the name we give to this accumulation of counting. Notice that we are always going to the right of the origin.

Example: $2+3$ means count the number of units from zero to 3 and total all these. The consecutive count is 5. Likewise $2+3$ means to count twice then count 3 more units and see where you land. Of course, you are 5 units from the origin. This is the concept of addition.

1.1.2 Subtraction

Notice that there are no numbers or concepts to the left of the origin.

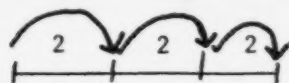


Thus, we can never count in that direction when dealing with the set of whole numbers. In order to subtract we can only count to the left and stop at zero. For example, $5-3$ means to start at the origin and count 5 to the right, the minus (-) means now to count 3 to the left. Notice that you stop at the number position 2.

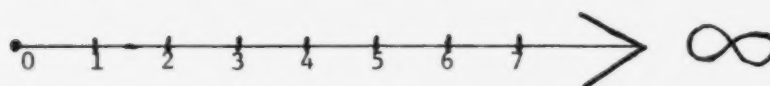
Now what is $2-6$? We count to the right 2 units, and then count to the left 6 units; however, we ran out of counting units at zero. Thus subtraction is "open" under the set of whole numbers in some cases. The concepts of "open" and "closed" systems are simple. If we say that subtraction is "open" for whole numbers, all we mean is that there will sometimes be no answer within the whole number system. "Closed" means that there is an answer. We will look at this situation later using a negative number line.

1.1.3 Multiplication

Multiplication is simply repeated addition. For example, 3×2 means count to the right two counts three times or 3×2 means $2 + 2 + 2$. Of course we land on the answer 6. We will always be to the right and thus multiplication is closed for the multiplication of whole numbers.



$$3 \times 2 = 6$$



1.1.4 Division

Division can be understood as a variation of addition. To demonstrate this think of any number as having a distance or length from the origin.

For example: Distance $\xrightarrow{6}$ = 2 ← number of "jumps"
 "Jump size" $\xrightarrow{3}$

represents how many "jumps" or counts of 'threes' I have to make to get from the origin to the number six.

If you make 3 counts and then 3 counts again you are at 6. Therefore we have to jump twice and our answer is 2 or $6/3 = 2$.

Take an example where it is not possible to arrive at a location in simple jumps. Example $\frac{5}{2} = ?$

How many counts of 'two's' will it take to get to 5. Well, let's see. $2+2$ is 4 all right. Now, $2+2+(2 \text{ again})$ is too far to the right; we can't land on 5. So division is "open" for some problems under the set of whole numbers.

Of course, you can see now why we need other sets of numbers including negative and fractions to complete the above operations.

PROBLEMS FOR MEDICAL LABORATORY TECHNICIAN PROGRAM

1.1 Whole Numbers

- (1) Pat Precise is a laboratory technician at Perpetual Care Clinic. One of the tasks every morning is to prepare blood collection trays for morning pick-ups. In each tray, Pat needs to put 12 purple-stoppered tubes, 20 red-stoppered tubes, 10 black-stoppered tubes and 6 grey-stoppered tubes. How many tubes does Pat place in each tray?
- _____
- (2) If Pat prepares four trays, how many tubes will be needed of each color? _____
How many tubes in all?
- _____
- (3) Pat is responsible for ordering the supply of blood collection tubes from the manufacturer. 100 tubes come in a box. Assuming all the tubes are used each day, how many boxes of purple-stoppered tubes will they need to order each week?
- _____
- (4) On Tuesday, while collecting blood, Pat used 17 red-stoppered tubes. How many was left in the tray?
- _____
- (5) On Monday, Pat used nine black tubes. On Tuesday, seven, on Wednesday seven, on Thursday only four. On Friday eight was used. What was the average number of tubes used per day?
- _____
- (6) One week, the tubes Pat had ordered did not come in. There was only 28 black-stoppered tubes left. How many were put in each tray?
- _____

PROBLEMS FOR MINING TECHNOLOGY PROGRAM1.1 Whole Numbers1.1.1 Addition

- (1) The three shuttle car operators of the 8-Main Section of Diamond Coal Co. reported that they ran 51, 52, and 53 cars respectively on the day shift. What was the total number of cars run on the 8-Main Section?
-
- (2) The three section foremen of Diamond Coal Company #1 Mine reported the tonnage recovered during the day shift was 1200, 1150, and 950 tons. What was the total tonnage recovered that day?
-

1.1.2 Subtraction

- (1) Kim, the foreman of 8-Main Section, recorded that 3 of the 10 miners did not report to work for the night shift. How many miners did report to work?
-
- (2) Marty, the mine foreman, discovered that an error had been made in the tonnage report. The total tonnage reported was 2350 tons. However, this figure was high by 420 tons. What was the correct tonnage?
-

1.1.3 Multiplication

- (1) If each shuttle car holds 7 tons and the 8-Main section had a total of 137 cars for the day shift, what was the tonnage for the section that shift?
-
- (2) If 10 persons work on each section of the Diamond #1 Mine and there are 5 sections, how many persons would work if the day shift and evening shift both worked one day?
-

1.1.4 Division

- (1) The total tonnage for Friday was 6475 tons. If each car carried 7 tons, how many cars were filled that day?
-
- (2) If 60 workers were divided evenly among five sections, how many workers would work on each section?
-

PROBLEMS FOR NURSING PROGRAM1.1 Whole Numbers

1.1.1 Addition

- (1) Calculate the intake of Patient X for 24 hours:

Breakfast: Coffee - 240 ml
 Juice - 120 ml
 Milk - 180 ml

Lunch: Soup - 100 ml
 Tea - 240 ml

3:00 p.m.: Water - 150 ml

Dinner: 7-Up - 100 ml
 Jello - 120 ml

8:00 p.m.: Ice
 Cream - 60 ml

9:00 p.m.: Water - 90 ml

Total - ____ ml

- (2) Calculate the output of Patient for 24 hours:

8:00 a.m.: Urine - 325 ml

11:00 a.m.: Liquid BM - 150 ml

12:00 a.m.: Urine - 400 ml

2:00 p.m.: Emesis - 200 ml

4:00 p.m.: Urine - 250 ml

8:00 p.m.: Urine - 300 ml

1:00 a.m.: Urine - 240 ml

3:00 a.m.: Emesis - 150 ml

5:00 a.m.: Urine - 200 ml

Total - ____ ml

1.1.2 Subtraction

- (1) At the beginning of your shift Patient X had 950 ml in his IV bottle. At the end of your shift the patient had 125 ml remaining. How many ml had infused?

- (2) Patient X's apical pulse is 84. Their radial pulse is 76. What is the pulse deficit?

PROBLEMS FOR NURSING PROGRAM CONTINUED

1.1.3 Multiplication

- (1) If Patient X takes Maalox 30 ml q 2 h, how many ml are needed for 24 hours?

- (2) Gantrisin is dispensed in doses of 15 grains. The doctor has ordered gantrisin for 5 different patients at 2 p.m. How many grains of gantrisin will be dispensed at 2 p.m. ?

1.1.4 Division

- (1) If 15 grains of gantrisin are available and the doctor has prescribed doses of 5 grains each for a number of patients, how many doses can be administered?

- (2) The IV order reads: administer 1000 ml. in 8 hours. How many ml. should the patient receive in 1 hour?

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM1.1 Whole Numbers

1.1.1 Addition

- (1) During the first week on the job at White Enterprises, Pat Blake filed the following documents:

Monday	157
Tuesday	168
Wednesday	97
Thursday	123
Friday	54

How many documents did Pat file the first week?

- (2) Pat entered the following checks in the company checkbook on Friday:

No. 415	\$ 75
No. 416	350
No. 417	59
No. 418	422
No. 419	62

What was the total amount of the checks Pat wrote on Friday?

1.1.2 Subtraction

- (1) White Enterprises keeps monthly sales records by department. Calculate the sales increase for each department for the month of December.

<u>Dept.</u>	<u>November Sales</u>	<u>December Sales</u>	<u>Amount of Increase</u>
A	\$15,726	\$16,321	_____
B	10,007	11,203	_____
C	14,695	17,014	_____
D	9,027	9,999	_____
E	8,546	9,052	_____

- (2) Inventory records of White Enterprises Finance Department revealed the following changes. Calculate the number of items that have been consumed for each category.

<u>Item</u>	<u>Beginning Inventory</u>	<u>Ending Inventory</u>	<u>Change</u>
File folders, 1/3 cut	18 boxes	13 boxes	_____
Index cards, 3 x 5	25 packs	18 packs	_____
Legal Pads, 14"	77 pads	42 pads	_____
Ball point pens, black	100 pens	57 pens	_____
Letterhead stationery	50 reams	19 reams	_____

PROBLEMS FOR SECRETARIAL ADMINISTRATIONS PROGRAM

1.1 Whole Numbers (continued)

1.1.3 Multiplication

- (1) Kim Haskins works as a temporary office worker for White Enterprises. Kim's hourly rate is \$4.00. Last week Kim worked eight hours on Monday; five hours on Tuesday; six hours on Wednesday; and eight hours on Thursday. How much did Kim earn?
-
- (2) Five college students work part-time during peak work loads at White Enterprises. Calculate the weekly gross earnings for each student.

<u>Student</u>	<u>Rate</u>	<u>Hours Worked</u>	<u>Gross Pay</u>
Student A	\$5.00	17	_____
Student B	4.00	23	_____
Student C	4.00	10	_____
Student D	5.00	20	_____
Student E	4.00	9	_____

1.1.4 Division

- (1) White Enterprises paid each of its workers at the end of the year. The amount for each department was distributed equally among the department's employees. Calculate the bonus per employee for each department.

<u>Department</u>	<u>Bonus</u>	<u>Employees</u>	<u>Share Per Employee</u>
Administration	\$25,000	5	_____
Sales	30,000	6	_____
Merchandising	16,000	8	_____
Finance	12,000	6	_____

- (2) Pat types at the rate of 60 words per minute, How long will it take Pat to type the following documents?

Document A	480 words	_____ minutes
Document B	640 words	_____ minutes
Document C	4,800 words	_____ minutes

BUSINESS MANAGEMENT

1.1.1 Addition

- (1) Apex Control manufactures heating elements for various industries.

Below are the number of units produced this year.

January.....23	July.....17
February.....27	August.....15
March.....42	September.....54
April.....40	October.....73
May.....33	November.....107
June.....21	December.....99

What is the total number of units produced? _____

- (2) Marty, a manager at Apex Controls, was assigned the task of finding the total cost of producing the heating elements. The following breakdowns are given:

Manufacturing Cost	\$3,630
Testing Cost	1,100
Packing Cost	275

What is the total cost of producing the elements? _____

1.1.2 Subtraction

- (1) Several of the heating elements were sold. Listed below are the companies, the list price, and the amount that each company actually paid for the elements. Determine how much less each company paid from that of the list price.

	LIST	AMT. PAID	AMT. SAVED
Company A	\$2,417	2,126	_____
Company B	1,237	1,088	_____
Company C	692	608	_____
Company D	855	752	_____

- (2) Apex sent word to their accounting department that they wanted to know how much the Owners Equity is. To find this, Pat the accountant, had to take the total Assets and subtract the total Liabilities from it.

Total Assets
\$70,000

Total Liabilities
\$21,500

Owners Equity? _____

BUSINESS MANAGEMENT (CONTD)

1.1.3 Multiplication

- (1) In May of this year Apex found out that it cost \$6.00 per unit to manufacture the heating elements. The number of elements manufactured in May were 33.

What is the total cost to make 33 units?

- (2) Kim was told that next year they would need to produce three times more units for January and July than this year.

This Year

January 23

July 17

How many total units must Kim produce next year for these two months?

1.1.4 Division

- (1) Last year Apex sold \$2,150 worth of elements at a cost of \$5.00 per unit.

How many units did Apex sell last year?

- (2) When the manufacturing line is running at full speed, Apex can produce six units per day. How many days would it take to produce the following if the line were running at full speed:

36 units	_____	days
72 units	_____	days
108 units	_____	days
120 units	_____	days

COMPUTER TECHNOLOGY

1.1.1 Addition

- (1) K.E.C. Data is a software company that writes personalized software for use in computer systems of different companies throughout the U.S.

Marty, a programmer, is setting up an array to hold all the phone numbers that the salespeople of Company X frequently call. Marty has been given the following information:

Salesperson A	12 Numbers	Salesperson E	55 Numbers
Salesperson B	27 Numbers	Salesperson F	9 Numbers
Salesperson C	43 Numbers	Salesperson G	4 Numbers
Salesperson C	13 Numbers	Salesperson H	39 Numbers

What is the total number of phone numbers that Marty must set the array up for? _____

- (2) After coding input statements for the following three categories, Kim was informed that more input statements must be added.

	<u>Coded Inputs</u>	<u>Added Inputs</u>
Sales	7	4
Adds	11	1
Rent	9	6

After Kim finished, how many total inputs were there?

1.1.2 Subtraction

- (1) One program written last year was able to take the taxes from the employees pay. Listed below are the employees, and their pay. Tell the amount that is left over after taxes.

Emp. A	\$ 456	Taxes	10.00	_____
Emp. B	976	Taxes	105.00	_____
Emp. C	1,237	Taxes	456.00	_____
Emp. D	1,765	Taxes	500.00	_____

- (2) One of the programs written for a bank would give the ending balance at the end of each month. This was set up in the program by taking the beginning balance and deducting the total amount of the checks for that month. Below is one example of such transaction. Find the amount at the end of the month (ending balance).

<u>Customer B</u>	<u>Beginning Balance</u>
<u>Checks for the Month</u>	
#301 \$1,234	\$4,932
#302 667	
#303 1,001	
What is the ending balance? _____	

COMPUTER TECHNOLOGY (CONTD)

1.1.3 Multiplication

- (1) In a program for determining pay, the hour field is multiplied by the rate of pay field to yield the pay.

Some of the data entered into the program was:

	<u>Hours Worked</u>	<u>Rate of Pay</u>	
A	40	\$4.00	_____
B	37	3.00	_____
C	22	5.00	_____
D	46	7.00	_____
E	2	9.00	_____

In the spaces provided, figure the pay for each employee.

- (2) A company wanted a program that would take their profit for one year and give them the amount of profit they would have for any given amount of years if the profit remained the same.

Big Company

Profit this year - \$14,500

Given years:

2	_____
5	_____
8	_____
10	_____

1.1.4 Division

- (1) A computer disk was able to hold 360K Bytes of memory.
How many programs could be put on this disk if the programs were 40K Bytes each? _____
- (2) If a printer were capable of printing 300 words per minute, then how many words are printed per second? _____

1.2 FRACTIONS

REVIEW NOTES

Fractions are examples of the natural mental concepts called "parts" of wholes. Imagine, for example, a quart glass fruit jar filled with 200 marbles and sealed. The concept now is one whole jar full of marbles. Now drop this "whole" off the front porch onto a flat rock. Suddenly the concept of fractions is made manifest. If you could only find 50 marbles scattered in the yard then you recovered a portion of 50 out of 200 or 50/200 which is $\frac{1}{4}$ of the whole.

The four operations of arithmetic (addition, subtraction, multiplication, and division) can be easily applied to fractions in a horizontal (across the page) format. This method is easily applied, always works, and has many applications in higher mathematics.

Fractions may be expressed as a number (numerator) over another number (denominator). If the numerator is the smaller number then the fraction is called proper. If the numerator is larger than the denominator then the fraction is called improper. When there is another whole number in front of a fraction then the expression is called mixed. For example:

$\frac{2}{3}$ Proper
Fraction

$\frac{3}{2}$ Improper
Fraction

$1\frac{1}{2}$ Mixed
Fraction

To convert a mixed expression to an improper fraction, multiply the denominator by the whole number and add this product to the numerator. Example:

$$3\frac{2}{5} = \frac{(5 \times 3) + 2}{5} = \frac{17}{5}$$

The first step in all fractional operations is to convert all mixed expressions into improper fractions. Do not change proper fractions.

1.2.1 Algorithm for Addition of Two Fractions

- A. Place the fractions next to each other, horizontally, side-by-side.
- B. Convert mixed expressions to improper fractions.
- C. Find a common denominator (some number that all denominators will divide into wholly). A method that always works is to simply multiply the denominators together.
- D. Divide the first denominator into the common denominator and multiply this by the first numerator.

- E. Bring down the plus sign.
- F. Divide the second denominator into the common denominator and multiply this by the second numerator.
- G. Add the numbers in the numerator.
- H. Reduce the fraction. This is accomplished by dividing the denominator into the numerator.

Example: Add $2 \frac{1}{3}$ and $\frac{4}{5}$

A. $2 \frac{1}{3} + \frac{4}{5}$

B. $\frac{7}{3} + \frac{4}{5}$

C. $\frac{\quad}{15}$

D. $\frac{35}{15}$

E. $\frac{35 + \quad}{15}$

F. $\frac{35 + 12}{15}$

G. $\frac{47}{15}$

H. $3 \frac{2}{15}$

1.2.2 Algorithm for Subtraction of Fractions

- A. Place the fractions next to each other, horizontally, side-by-side.
- B. Convert mixed expressions to improper fractions
- C. Find a common denominator (some number that all denominators will divide into wholly). A method that always works is to simply multiply the denominators together.
- D. Divide the first denominator into the common denominator and multiply this by the first numerator.
- E. Bring down the minus sign.
- F. Divide the second denominator into the common denominator and multiply this by the second numerator.
- H. Reduce the fraction. This is accomplished by dividing the denominator into the numerator.

1.2.3 Algorithm for Multiplication of Fractions

- A. Place the fractions adjacent to each other, horizontally, side-by-side with multiplication sign between them.
- B. Convert mixed expressions to improper fractions.
- C. Multiply straight across, numerators by numerators and denominators by denominators.
- D. Place the product of numerators over the product of denominators.
- E. Reduce, if possible.

Example: Multiple $2 \frac{1}{3}$ by $\frac{4}{5}$

A. $2 \frac{1}{3} \times \frac{4}{5}$

B. $\frac{7}{3} \times \frac{4}{5}$ (Example of converting mixed expressions to improper fractions: $2 \frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$)

C. $\frac{28}{15}$

D. $1 \frac{13}{15}$

1.2.4 Algorithm for Division of Fractions

- A. Place the fractions adjacent to each other horizontally, side-by-side with a division sign between them.
- B. Convert all mixed expressions to improper fractions.
- C. Invert (turn over, reversing the numerator and denominator) the second fraction and change the division symbol to multiplication.
- D. Multiply straight across, placing the product of the numerators over the product of the denominators.
- E. Reduce, if possible.

Example: Divide $2 \frac{1}{3}$ by $\frac{4}{5}$

A. $2 \frac{1}{3} \div \frac{4}{5}$

B. $\frac{7}{3} \div \frac{4}{5}$

C. $\frac{7}{3} \times \frac{5}{4}$

D. $\frac{35}{12}$

E. $2 \frac{11}{12}$

PROBLEMS FOR MEDICAL LABORATORY TECHNICIAN PROGRAM1.2 Fractions

- (1) One Saturday, the regular technician who does urinalysis, called in sick. There are 42 urine specimens that day. Pat Precise does 14 of them. What fraction of the urine does Pat analyze: (Reduce to lowest value.)
-
- (2) Employee X, who works in the chemistry lab, also does 14 urines. Pat's supervisor analyzes seven urine specimens. What fraction of the urine workload do they do together?
-
- (3) No one is able to finish the remaining urines which Pat, Chris and the supervisor have not had time to do. When they leave, what fraction of the urines are left for the afternoon shift?
-
- (4) Pat and Chris are making a series of dilutions for a serology test. Their first dilution is $\frac{1}{4}$ and the second dilution is $\frac{1}{2}$. They know that they must multiple to arrive at their dilution. What final dilution have they made?
-
- (5) If they make a further $\frac{1}{2}$ dilution, what is their final dilution?
-

PROBLEMS FOR MINING TECHNOLOGY PROGRAM1.2 Fractions1.2.1 Addition

- (1) The following amounts of cuts were made at the 5 sections of Diamond #1 Mine on the day shift: $6 \frac{1}{2}$, 8, $7 \frac{1}{2}$, $6 \frac{1}{2}$. How many cuts were made?
-

- (2) The miners on 8-Main Section worked the following amount of overtime this week: $2 \frac{1}{4}$, $3 \frac{1}{2}$, $2 \frac{1}{2}$, 5, $5 \frac{1}{2}$, $7 \frac{1}{4}$, $6 \frac{3}{4}$, and $9 \frac{1}{2}$. What was the total hours of overtime worked?
-

1.2.2 Subtraction

- (1) If the total hours of overtime worked was $20 \frac{1}{2}$ and Lee worked $3 \frac{3}{4}$ of these hours, how many hours did the rest of the miners work?
-

- (2) The height of the coal on 8-Main Section was $4 \frac{1}{2}$ feet, but overnight the roof squeezed down $\frac{2}{3}$ of a foot. What is the height of the coal now?
-

1.2.3 Multiplication

- (1) $2 \frac{3}{4}$ of the 160 miners at Diamond #1 Mine went on strike. How many miners are on strike?
-

- (2) A flu epidemic caused $\frac{1}{3}$ of the 210 miners at Diamond #2 Mine to miss work. How many miners missed work?
-

1.2.4 Division

- (1) If the total tonnage recovered at 8-Main Section was 1100 tons and the mines made $6 \frac{1}{2}$ cuts, how many tons were recovered per cut?
-

- (2) If $25 \frac{1}{2}$ hours overtime were divided among 4 miners, how many extra hours did each person work?
-

PROBLEMS FOR NURSING PROGRAM1.2 Fractions1.2.1 Addition

- (1) Consolidate the contents of the following three bottles into one bottle and label the resulting quantity appropriately: $2 \frac{1}{4}$ ounces, $3 \frac{1}{8}$ ounces, and $1 \frac{1}{2}$ ounces. What would be the total ounces in one bottle?
-
- (2) If the baby had drunk $\frac{2}{3}$ ounces and $\frac{3}{5}$ ounces, what would the total consumption have been?
-

1.2.2 Subtraction

- (1) You are completing an inventory of stock solutions. There had been 32 ounces of boric acid, $3 \frac{5}{8}$ ounces have been used up. How much is left?
-
- (2) You have $\frac{3}{4}$ ml. of a drug in a vial. You are to give an injection of $\frac{1}{2}$ ml. How much of the drug will remain in the vial when you are finished?
-

1.2.3 Multiplication

- (1) You are helping Patient X figure caloric intake for the day. The patient had a box of cereal containing $\frac{7}{8}$ ounces. The label states that there are 112 calories in one ounce of cereal. How many calories are there in the box of cereal?
-
- (2) In order to calculate the IV rate you need to know: how many minutes there are in $5 \frac{1}{4}$ hours?
-

1.2.4 Division

- (1) If Patient X has eight tablets and must take $\frac{1}{2}$ tablet a day, how many days should their supply last?
-
- (2) If a nurse's aide has nine inches of adhesive tape and cuts the tape into pieces of $\frac{1}{3}$ inch, how many pieces of tape will they have?
-

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM

1.2 Fractions

1.2.1 Addition

- (1) Jones, a salesperson for White Enterprises, worked the following hours over a two-week period. Calculate the total hours worked each week and the total for the two-week period.

	<u>Monday</u>	<u>Tuesday</u>	<u>Wednesday</u>	<u>Thursday</u>	<u>Friday</u>	<u>Total</u>
Week 1	8 1/2	6 3/4	7 3/4	8 1/4	9 1/2	_____
Week 2	8 1/4	8 3/4	7 1/2	8 3/4	8 1/2	_____
				Grand Total		_____

- (2) During the past week salespersons for White Enterprises, used following amounts of gasoline. Calculate the total amount of gas they used for the week.

Monday	40 1/2 gallons
Tuesday	14 3/8 gallons
Wednesday	25 1/4 gallons
Thursday	16 7/8 gallons
Friday	27 1/4 gallons

Total amount of gas used _____

1.2.2. Subtraction

- (1) Below is a list of stocks owned by White Enterprises. Their opening and closing prices for June 15 are given. Calculate the amount that each stock fell on June 15.

<u>Stock</u>	<u>Opening Price</u>	<u>Closing Price</u>	<u>Amount of Change</u>
Stock A	17 1/2	12 3/9	_____
Stock B	78 3/4	70 1/4	_____
Stock C	29 1/4	27 1/2	_____
Stock D	39 1/2	30 3/4	_____
Stock E	19 1/4	15 3/4	_____

- (2) March Bakery, a subsidiary of White Enterprises, recorded the following amounts of sugar on hand at the beginning and end of the day last week. Calculate the amount of sugar that was used each day.

	<u>Morning</u>	<u>Night</u>	<u>Amount of Change</u>
June 15	726 1/2 lbs	701 3/4 lbs	_____
16	701 3/4 lbs	615 1/8 lbs	_____
17	615 1/8 lbs	543 1/2 lbs	_____
18	543 1/2 lbs	400 1/4 lbs	_____
19	400 1/4 lbs	352 1/8 lbs	_____

1.2.3 Multiplication

- (1) Lee Jacobson works as a clerk/cashier at the March Bakery. Lee's hourly rate is \$3.85. Calculate Lee's earnings last week.

<u>Date</u>	<u>Hours</u>	<u>Earnings</u>
Feb. 7	6 $\frac{3}{4}$ hours	_____
Feb. 8	7 $\frac{1}{2}$ hours	_____
Feb. 9	7 $\frac{3}{4}$ hours	_____
Feb. 10	5 $\frac{1}{2}$ hours	_____
Feb. 11	7 $\frac{1}{4}$ hours	_____

- (2) David Enterprises paid \$750 an acre for a $15 \frac{3}{4}$ acre tract. What was the total cost of the land?
- _____

1.2.4 Division

- (1) Pat Templeton is a driver for David Enterprises. Last week Pat made two trips in the times listed. What speed did Pat average for each trip?

			<u>Average</u>
Monday	549 $\frac{3}{4}$ miles	9 $\frac{3}{4}$ hours	_____
Wednesday	323 $\frac{1}{2}$ miles	6 $\frac{1}{4}$ hours	_____

- (2) Lee Blake and Pat Sutherland attended a three-day convention of Professional Secretaries International. They divided expenses as indicated. Calculate each one's share of the lodging and travel expense.

Lodging	\$185.00	Lee - $\frac{3}{4}$ of total	Pat - $\frac{1}{4}$ of total
Travel	360.00	Lee - $\frac{1}{3}$ of total	Pat - $\frac{2}{3}$ of total

_____	_____
_____	_____

BUSINESS MANAGEMENT

1.2.1 Addition

- (1) Chris, the personnel manager at Apex, was told to cut back on supervisors overtime. To do this Chris wanted to find out the hours that each of the two supervisors worked in a given week. From the timecards Chris obtained the information below. How many hours did each supervisor work for this week?

	<u>Mon.</u>	<u>Tue.</u>	<u>Wed.</u>	<u>Thur.</u>	<u>Fri.</u>	<u>Totals</u>
#1	8 3/4	8 1/2	9	7 3/4	9 1/2	_____
#2	6 1/2	9 3/4	7 3/4	9	8 3/4	_____

- (2) To meet production quotas a certain amount of work hours are needed for each product every day. How many total work hours are needed per day?

Heating Elements	18 1/2
Thermostats	20 3/4
Switching Division	6 1/2
Magnetic Control	50 7/8
Overload Protector	10 5/8

1.2.2 Subtraction

- (1) Apex uses 18 gauge wire in the making of several electrical components. The list below contains the amount of wire on hand at the beginning and end of the day. Calculate the amount of wire that was used each day.

Day	Beginning	End	Amount Used
Monday	940 1/2	912 1/4	_____
Tuesday	912 1/4	887 5/8	_____
Wednesday	887 5/8	840 3/8	_____
Thursday	840 3/8	799 1/4	_____
Friday	799 1/4	755 3/4	_____

- (2) Lee had previously found that it took 31 3/8 hours to do a certain job. This week Apex received a new piece of machinery that would reduce the needed hours by 12 1/4 hours. How many hours would it now take to do the job? _____

BUSINESS MANAGEMENT (CONTD)

1.2.3 Multiplication

- (1) $\frac{3}{8}$ of an eight hour work day is spent on making heating elements. How many hours are spent each day on making heating elements?
-

- (2) If $\frac{1}{4}$ of the 360 workers on first shift belonged to the company credit union, how many first shift workers would belong?
-

1.2.4 Division

- (1) If there were a total of 140 heating elements made in $3\frac{1}{2}$ work shifts, how many elements were made per shift?
-

- (2) If you had 20 inches of twenty-four gauge wire and you needed it cut into $\frac{1}{4}$ inch pieces, how many pieces would you have?
-

COMPUTER TECHNOLOGY

Francements

1.2.1 Addition

- (1) In writing a particular piece of software, it was determined that the following times would be needed.

Documentation	3 3/4 hours
Programming	40 5/8 hours
De-bugging	16 2/3 hours

How many total hours are needed to complete the software?

- (2) If a program took $12 \frac{3}{4}$ seconds to boot up and then the title page appeared for $10 \frac{5}{7}$ seconds before the menu came up, how many total seconds did it take before you saw the menu?

1.2.2 Subtraction

- (1) The speed of a particular calculation in a program took $2 \frac{3}{4}$ seconds. The programmers analyzed the coding and found they could reduce the time it took by $1 \frac{3}{5}$ seconds. How long would it now take to perform the calculation?

1.3 DECIMALS

REVIEW NOTES

Decimals are a systematic way of expressing portions of amounts. A convenient reference base of ten is used to make quantities understandable. Given the decimal location one must first know the place value of each place holder in a numeric expression as indicated below.

Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths	Ten Thousandths
$\frac{1}{10000}$	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{1}$	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$

1.3.1 Algorithm for Adding Decimal Numbers

- A. Place decimal numbers in a vertical format (one underneath the other) keeping the decimal point straight.
- B. Add values and carry higher unit values to next column just as in adding whole numbers. Keep the decimal in its location.

Example: Add 33.33 and 444.4

A.
$$\begin{array}{r} 33.33 \\ 444.4 \\ \hline \end{array}$$
 (note that decimals are lined up.)

B.
$$\begin{array}{r} 33.33 \\ 444.40 \\ \hline 477.73 \end{array}$$
 (Add values, providing zeros to the right of the decimal - of course, this does not change the number.)

1.3.2 Algorithm for Subtracting Decimal Numbers

- A. Place decimal numbers in a vertical format (one underneath the other) keeping the decimal point straight.
- B. Subtract values and borrow from higher unit values as needed just as in subtracting whole numbers. Keep the decimal straight.

1.3.3 Algorithm for Multiplying Decimal Numbers

- A. Place the decimal numbers in a vertical format.
- B. Multiply the digits as you would whole numbers ignoring the decimal point.
- C. Write down the product of the multiplied digits.
- D. Add the number of decimal places in your original numbers, counting any zeros you added in A.
- E. Move the decimal that number of places from the right hand side of your answer over toward the left.

Example: Multiply 1.7 times 1.31

- A. 1.70 (Add zeros to right of decimal without changing value)

$$\begin{array}{r}
 \text{B.} \quad 1.70 \\
 \quad 1.31 \\
 \hline
 \quad 170 \\
 \quad 510 \\
 \hline
 \text{C.} \quad 22270
 \end{array}$$

- D. Four (4) decimal places, counting the zero added in A.
- E. 22270 = 2.2270 or 2.277 (move the decimal four places)

1.3.4 Algorithm for Dividing Decimal Numbers

To apply the following algorithm, you will need to remember some terms and one simple rule. Otherwise, the process is the same as for division of whole numbers.

Terms: Divisor - the number you divide by

Divident - the number you divide into

Quotient - the result

Rule: In dividing decimal numbers, you must convert the divisor into a whole number and adjust the dividned accordingly.

- A. Set up the problem with a division bracket as follows:

$$\begin{array}{r}
 \text{Quotient} \\
 \hline
 \text{Divisor} \overline{) \text{Dividend}}
 \end{array}$$

- B. Apply the rule so that any decimals in the divisor are moved to the right.

- C. Adjust the dividend by moving the decimal the same number of places to the right. You may have to add zeros to the right of the original decimal point.
- D. Place a decimal on the bracket (in what will be the quotient) directly above the resulting decimal in the dividend. You may have to add more zeros after the new location so that the quotient can be determined to two (2) decimal points.
- E. Divide as you would with whole numbers and the result (quotient) will have the decimal located correctly.

Example: Divide 31.1 by 1.24

A. $1.24 \overline{) 31.1}$ (Be careful to set up problem correctly.)

B. $1.24 \overline{) 31.1}$ (Move the decimal in the divisor.)

C. $124 \overline{) 31.10}$ (You have to add a zero here because it would not be possible to move the decimal the same number of places in the dividend.)

D. $124 \overline{) 3110.00}$ (Zeros added to provide two decimal places in quotient.)

E. $124 \overline{) 3110.00}$

$$\begin{array}{r}
 25.08 \\
 124 \overline{) 3110.00} \\
 \underline{248} \\
 630 \\
 \underline{620} \\
 1000 \\
 \underline{992} \\
 8
 \end{array}$$

PROBLEMS FOR MEDICAL LABORATORY TECHNICIAN PROGRAM1.3 Decimals

- (1) As Pat Precise works in chemistry, a manual procedure must be done. Into each tube Pat puts 5.5 ml of Reagent A, 0.1 ml of Reagent B. How many ml are in each tube?

- (2) When you began your procedure, you had 100 ml of Reagent A. You used it for seven tubes. How much Reagent A is left?

- (3) In reading their test, Pat's sample has an absorbance of .539. The serum blank has an absorbance of .008. What is the difference of the two absorbances?

- (4) In the hematology lab, you are doing a series of hemoglobins. You notice that one patient's value is 11.2 g/dl (grams/deciliter). You know that the previous day it had been 13.9 g/dl. How much had the hemoglobin decreased?

- (5) You had 2.2 hours of overtime. You were paid \$25.74. What was your hourly rate for the overtime?

- (6) If Pat had worked 40 hours at a regular rate of \$7.80 per hour and had 3.7 hours of overtime, what would the total pay have been?

PROBLEMS FOR MINING TECHNOLOGY PROGRAM

1.3 Addition

- (1) The following amounts of overtime pay were reported paid to the workers at 8-Main Section: \$16.25, \$50.35, \$86.75 and \$90.10. What was the total amount of overtime pay?
-
- (2) The five sections of Diamond #1 Mine reported the following number of cuts made that day: 6.5, 7.5, 8, 5.5, and 4. What was the total amount of cuts made?
-

1.3.2 Subtraction

- (1) The warehouse manager cut five pieces of chain from a 100 foot roll. The pieces measured 2.5, 3.5, 20, and 6.5 feet. How much chain is left?
-
- (2) The maintenance foreman pumped 10.6 gallons of hydraulic oil into a continuous miner. How much oil is left if he pumped it from a full 55 gallon drum?
-

1.3.3 Multiplication

- (1) Six workers shoveled 2.4 tons of coal each. How much coal was shoveled?
-
- (2) The six workers who shoveled coal were rewarded with 2.5 hours overtime each. How much overtime pay was paid to each worker if the overtime pay is \$15 per hour?
-

1.3.4 Division

- (1) 2.8 miles of track had to be cleared by three workers. How much did each miner receive if all four received the same amount of money?
-

PROBLEMS FOR NURSING PROGRAM1.3 Decimals1.3.1 Addition

- (1) The nurse needed three different solutions into which was poured the following quantities of atropine: 0.052 milligrams, 0.8 milligrams, and 0.12 milligrams. What was the total amount of atropine used?
-
- (2) Patient X's heart is in a weakened condition. The patient has been given two doses of a 1/1000 solution of epinephrine as a stimulant. The first dose given was 0.03 milliliters and the second dose was 0.045 milliliters. How many milliliters has the patient received so far?
-

1.3.2 Subtraction

- (1) An infant measured 19.75 inches at birth. One month later, the infant was 21.25 inches. How much had the infant grown?
-
- (2) Initially, a patient received an oral dose of 1.5 milligrams of his medicine. In the next three days, they received 0.5 milligrams, 0.4 milligrams. How much is left from the original supply of 3.5 milligrams?
-

1.3.3 Multiplication

- (1) You have 71.3 grams of powder and must make up 10 doses from this supply. How many grams would you put in each dose?
-
- (2) Chlormerodrin is available in 18.3 milligram tablets. The doctor prescribes four tablets daily. How many milligrams of chlormerodrin is the patient receiving daily?
-

1.3.4 Division

- (1) The physician has ordered digoxin 0.125 mg. On hand are tablets labeled 0.25 mg. How many tablets should the nurse administer?
-

- (2) The physician has ordered prednisone 10 mg. to be given orally. The tablets available are labeled 2.5 mg. How many tablets should you give?
-

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM1.3 Decimals1.3.1 Addition

- (1) Chris recorded the following equipment expenses for David Enterprises last year. Calculate the total equipment expense.

Typewriters	\$ 2,726.27
Calculators	514.90
File Cabinets	420.48
Printers	4,106.20
Computers	<u>12,146.75</u>

Total expense \$ _____

- (2) Salesmen for White Enterprise logged the following miles last week. Chris was asked to calculate the total. What answer should Chris get?

Monday, 238.6 miles; Tuesday, 319.7 miles; Wednesday, 197.5 miles; Thursday, 333.7 miles; Friday, 215.8 miles.

Total miles _____

1.3.2 Subtraction

- (1) The bank statement of March Bakery showed \$5,276.28. A check for \$317.18 was outstanding. What will the balance be after the check clears the bank?

Balance after check clears _____

- (2) Chains of the White Enterprises firm showed the following gross sales and sales returns on Monday. Calculate the net sales for each store.

	<u>Gross Sales</u>	<u>Sales Returns</u>	<u>Net Sales</u>
Store A	\$5327.17	\$227.23	_____
Store B	3,006.42	146.80	_____
Store C	2,469.98	302.09	_____

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM (CONTD)

1.3.3 Multiplication

- (1) Pat keeps a record of the gas consumed by salespersons cars and the amount of tax paid on the gas. Calculate the total amount of tax paid to each state for the following figures.

	<u>Tax Rate</u> <u>Per Gallon</u>	<u>Gasoline Consumed</u>	<u>Total Tax</u> <u>Paid</u>
State A	10.3 ¢	117 gallons	_____
State B	9.7 ¢	208 gallons	_____
State C	11.2 ¢	93 gallons	_____

- (2) Pat's supervisor asked Pat to extend the following invoice. What figures should Pat have in the column labeled "Extension"?

<u>Item</u>	<u>Quantity</u>	<u>Price</u>	<u>Extension</u>
A	42	\$ 4.17	_____
B	112	.89	_____
C	18	8.75	_____
D	355	7.46	_____
E	62	12.50	_____

1.3.4 Division

- (1) Chris Bent, a security guard for White Enterprises, earns \$205.85 per week for a 40-hour week. How much does earn per hour?

- (2) White Enterprises owes a balance of \$11,420.75 which they will pay in 36 months. What will the monthly payments be?

BUSINESS MANAGEMENT

1.3.1 Addition

- (1) The following cost were occurred in one day of operation:

Electricity	\$ 48.72
Gas	121.46
Water	5.09
Wire	499.30

What is the total cost that occurred on this day?

- (2) Below is the amount of mercury purchased in July for thermostate.

<u>Date</u>	<u>Amount</u>
02	21.67 liters
10	12.04 liters
25	31.99 liters

What is the total amount of liters purchased for July?

1.3.2 Subtraction

- (1) If we bought a spool of wire for \$872.61 and we were given a discount of \$87.26, how much did we pay for the wire?
-

- (2) If we had a total of \$967.14 in petty cash and the following purchases were made:

Stamps	\$48.66
Paper	24.97
Envelopes	17.20

How much money is left in petty cash?

1.3.3 Multiplication

- (1) We bought some stock in another company. Below is the amount and the price per share.

<u>Amount</u>	<u>Price Per Share</u>
50 shares	\$4.59

How much did the stock cost us? _____

BUSINESS MANAGEMENT (COND)

Multiplication (Contd)

- (2) If .09 liters of mercury goes in each thermostat we make, how many liters of mercury does it take for 21 thermostats?
- _____

1.3.4 Division

- (1) If a customer purchased \$21, 407.92 worth of products from us, and we agreeded to let them pay for it in 12 installments, how much would each installment be?

\$ _____

- (2) If an employee earns \$182.80 for a 40 hour work week, how much do they make per hour?

\$ _____

COMPUTER TECHNOLOGY

Decimals

1.3.1 Addition

- (1) In figuring this months bill for access time to C B C Database Systems, we needed to know the total time accessed.

<u>Day</u>	<u>Time</u>
Monday - 07	13.67 Minutes
Friday - 11	26.03 Minutes
Thursday - 17	5.82 Minutes
Wednesday - 23	14.00 Minutes

What is the total amount of time accessed?

- (2) One of our programs would add the value of π (3.1415), to a given number. Tell the results.

<u>π</u>	<u>Number added to</u>	<u>Result</u>
3.1415	6.2719	_____
	13.003	_____
	0.6671	_____
	21.0902	_____

1.3.2 Subtraction

- (1.) If the CPU in computer A took 12.3 MS to process some data and computer B took 5.87 MS to process the same data, how much less time did computer B take?
- _____

- (2.) Our current printer can print 300 words in 1.0877 seconds. We are looking at a new printer that can print 300 words in 0.998 seconds. How much faster is the new printer?
- _____ seconds

COMPUTER TECHNOLOGY (CONTD)Decimals

1.3.3 Multiplication

- (1) We have a program that calculates an employees pay by multiplying the rate per hour field by the hours worked field. Tell the pay for each of the following:

<u>Employee</u>	<u>Rate Per Hour</u>	<u>Hours Worked</u>	<u>Pay</u>
A	\$3.35	40.2	_____
B	5.16	26.08	_____
C	8.99	42.33	_____

- (2) If our computer can retrieve data at a rate of 21.6 K Bytes per second, how much data would be retrieved in 7 seconds?

1.3.4 Division

- (1) If it took 83.6 hours to write a program and the total cost of the program was \$1254.00, how much did the programming cost per hour?

\$ _____

- (2) If one of our programs made \$14,300.67 in one year, then how much was it making each month?

\$ _____

1.4 RATIO AND PROPORTION

REVIEW NOTES

- 1.4.1 Ratios are simply comparisons by the arithmetic operation division of some given quantities. For example, to find the ratio of males to females on the campus of Southeast Community College, we divide the number quantity of males by the number quantity of females. In other words, males are the numerator and females are the denominator in this case. Suppose there were 139 males and 278 females. Then the ratio would be $\frac{139}{278}$ reduced

to $\frac{1}{2}$, or the ratio of males: females is 0.5:1, read as males to females is one half to one. Of course, the ratio of females to males would be 2:1.

1.4.2 Techniques

A simple method to remember is that the noun following (to) will always be the denominator. In the example above, note that the word problem states "...the ratio of males to females." Sometimes an expression of unit cost uses the word "per". For example, "What's the cost per ton of coal today"? Would be analyzed as follows: the quantity of coal (following per) would become the denominator. If three tons of coal costs \$70.50, what is the cost per ton? Ok, now create a ratio: $\frac{70.50}{3}$ reduces to $\frac{23.50}{1}$

or the ratio of costs to tonnage is \$23.50: 1.

1.4.3 Algorithm for Constructing Ratios

- A. Read the sentence(s).
- B. Determine the values for comparison.
- C. Determine the value to be placed in the numerator and that of the denominator.
- D. Express as a fraction and reduce.

Example: Find the ratio of Republicans to Democrats in the Freshman class. There are 200 Republicans and 100 Democrats.

- A. Read the sentence(s).
- B. Comparing 200 Republicans to 100 Democrats
- C. $\frac{200 \text{ Republicans}}{100 \text{ Democrats}}$
- D. $\frac{200}{100} = \frac{2}{1}$

1.4.4 Algorithm for Constructing Proportions

Proportion is a very similar concept to that of ratio. There are two important differences:

- (1) Proportions are expressed as decimals rather than as fractions.
- (2) Proportions are usually expressed as a part of a whole. That is, proportions are a special type of ratio taking the following form: $\frac{\text{part}}{\text{whole}}$

Notice that ratios are more often constructed to compare one part to another part. In the ratio example on the previous page, we were comparing the Republican part of the freshman class to the Democrat part of the freshman class.

- A. Carefully read the sentences determining the part to be compared to the whole.
- B. Determine the quantity value for the whole (note: you may have to add up all the parts in order to do this).
- C. Form a ratio of the part to the whole.
- D. Use decimal division (See 1.3.4 if you need help) dividing the whole into the part. That is, the whole will be your divisor and the part will be your dividend. The proportion will be your quotient.

Example: Find the proportion of Democrats in the Freshman class, using the data from 1.4.2:

- A. Read sentences carefully.
- B. 300 = whole (assuming all freshmen are either Republicans or Democrats, just add the two parts: 200 + 100 = 300.)

$$C. \frac{\text{Part}}{\text{Whole}} = \frac{\text{Democrats}}{\text{Whole}} = \frac{100}{300} = \frac{\text{Dividend}}{\text{Divisor}}$$

$$D. \begin{array}{r} 300 \overline{) 100.000} \\ \underline{900} \\ 1000 \\ \underline{900} \\ 1000 \\ \underline{900} \\ \text{etc.} \end{array} \quad \text{Quotient is Proportion}$$

As you will see in the next session (1.5 Percent) the percentage is simply found by multiplying the proportion times 100.

PROBLEMS FOR MEDICAL LABORATORY TECHNICIAN PROGRAM1.4 Ratio

- (1) Pat Precise is setting up a test which uses 1 ml of serum and 4 ml of reagent. What is the ratio of serum to reagent?
-

- (2) The automated blood counter at Perpetual Care Clinic is out of order so Pat must do blood cell counts manually. To do a white blood cell count, Pat uses .5 parts of blood and dilutes it to ten total parts. What is the ratio of undiluted to diluted blood?
-

PROBLEMS FOR MINING TECHNOLOGY PROGRAM1.4 Ratio

1.4.3 As Percents

- (1) The ration of people who missed work to people who worked was 1:8. What percent of the people worked?
-

- (2) The ratio of people working to tons produced was 8:1200. What percent of the total coal did each person produce?
-

PROBLEMS FOR NURSING PROGRAM1.4 Ratio

1.4.2 Techniques

- (1) Lee weighs 30 pounds. The average adult weighs 150 pounds. What is the ratio of Lee's weight to the average adult?

- (2) Lee's mother gives Lee aspirin in four equal parts. What is the ratio of one of the small pieces to the whole aspirin?

1.4.3. As Percents

- (1) About 25% of nursing students decide upon their careers before they are 10 years old. The ratio of these students to all nursing students is

- (2) In a 1:20 solution, there is one ounce of medication in 20 ounces of solution. What is the percentage of medication in the solution?

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM1.4 Ratios1.4.1 Definitions

- (1) When two numbers are compared by division, the division is called a ratio.

20 $\div$ 5 is an example of a

_____.

- (2) To find a rate, the numerator is divided by the denominator. In the rate 30 $\div$ 3 = 10, 30 is the

_____.

1.4.2 Techniques

- (1) Marty can type 300 words in 5 minutes. How many words can Marty type in one minute.

_____.

- (2) Marty earns \$200 per week. How many weeks would it take Marty to earn 800 ?

_____.

1.4.3 As Percents

- (1) White Enterprises recently ran 3 advertisements in the Daily News, and they averaged 45 sales as a result of the ads. What was the average percent of sales for the advertisements?

_____.

- (2) Receipts from 45 sales that were made as a result of 3 advertisements totaled \$15,000. The total cost of the ads was \$600. What was the average percent of sales?

_____.

PROBLEMS FOR BUSINESS MANAGEMENT1.4 Ratios

- (1) We have one employee with a college degree to every five employees without a college degree. What is the ratio of employees with degrees to those without?
- _____
- (2) The ratio of employees hurt on the job to those not hurt is 1:9. What percent of the employees are not hurt on the job?
- _____
- (3) An employee named Lee produces five heating units a day. The average employee produces 15 heating units per day. What is the ratio of Lee's production to that of the average employee?
- _____

PROBLEMS FOR COMPUTER TECHNOLOGY1.4 Ratio

- (1) We want to find the ratio of programs written for individual owned companies to corporations. There are 28 programs written per year for individual owned companies and 112 programs written per year for corporations. What is the ratio?
-
- (2) Three hours of debugging is spent for every 27 hours of programming. What is the ratio of debugging time to programming time?
-
- (3) About 25% of all our programers have their masters degree before they are 30 years of age. The ratio of programers with Masters degrees to all programers is?
-

1.5 PERCENTS

REVIEW NOTES

Percent expresses the mental concept of parts or portions based on 100 as a whole unit. If you think of a bucket of water filled as 100% full, then when it is $\frac{1}{4}$ full, you have $\frac{1}{4}$ of 100% or 25%.

Since percent is based as 100% where 1 = the whole, then all parts are expressed as hundreths, which is the second place holder to the right of the decimal. $\frac{1}{4}$ is $\frac{25}{100}$ or in decimal form

(.25) which is 25%.

You may already recognize the root word "cent" which refers to one hundred (centennial means 100 years and there are 100 cents in a whole dollar). Thus, percent really is best understood as a "per one hundred." Percent is really like a "standard" ratio that we are all accustomed to using. When we have to compare parts of things, it is helpful to convert them to the "standard" language of percentages. For example, it is much easier to compare 25% to 33% than it is $\frac{1}{4}$ to $\frac{1}{3}$.

1.5.1 Algorithm for Finding the Quantity Equal to a Certain Percent (Example: What is 23.75% of 80?)

- A. Change the percent expression to a decimal by moving the existing decimal two places to the right and drop the percent sign. Example: $23.75\% = .2375$;
 $24\% = .24$ (=) *
- B. Multiply the decimal form of the percent by the whole quantity. For example, 24% of 80
 $.24 \times 80 = 19.2$
- C. The product is the answer.
* = is read as "...approximately equal to." Of course, we are simply rounding off the number.

1.5.2 Algorithm for Unknown Percents

Unknown percents are of several forms. Examples are as follows:

A is what percent of B or what percent of B is A.

- A. Change to a ratio. The quantity after (of) is the denominator $\frac{A}{B}$
- B. Divide and obtain a decimal form.
- C. Change to a percent form by moving decimal and adding a percent sign.

Example: 2 is what percent of 10.

A. $\frac{2}{10}$

B. .2

C. 20%

1.5.3 Percent Reduction

Percent reduction means reducing or subtracting a given percent of a value from the original value. Format is usually as follows:

- A. Change percent to a decimal expression.
- B. Multiply decimal expression by amount to be reduced and get a product.
- C. Subtract this product from original amount to be reduced; this is your answer.

Example: Reduce 26.50 by 20%.

A. 20% is .2

B. $.2 \times 26.50 = 5.300$ (product)

C. $26.50 - 5.3 = 21.20$

PROBLEMS FOR MEDICAL LABORATORY TECHNICAIN PROGRAM

1.5 Percents

Problems 1,2, and 3 are related:

- (1) Pat Precise works in hematology doing differential white blood cell count. Pat looks at the cells under the microscope and counts 100 cells. Of these, 72 are identified as neutrophils. What percentage of the white blood cells are neutrophils?
-

- (2) Pat finds that the patient's total number of white blood cells is 9,000/ml (microliter). How many of the cells in one microliter are neutrophils?
-

- (3) In doing white blood cell counts, any cell with a nucleus is counted. Pat notices that 2 out of every 100 cells are really nucleated red blood cells. How many of the total cells are red blood cells?
-

Problems 4 and 5 are related;

- (4) Next Pat takes an inventory of the units of blood in the blood bank. Pat finds 20 group O, 17 groups A, 9 group B, and 4 group AB. What percentage of each group of blood does she have?
-

- (5) Pat sends 4 group O and 2 group A units back to the blood center. By what percentage has Pat reduced her total number of units?
-

- (6) Last week Pat ordered 30 Rh positive units and 10 Rh negative units of blood. The supervisor, Chris Supertech, tells Pat to increase the order by 30%. How many units of each should be ordered?
-

- (7) While processing units of blood, Pat must make a 5% solution of red blood cells (rbc). If Pat makes 100 ml of solution, how many ml of rbc will be needed?
-

- (8) Pat is in hematology again, and is doing a reticulocyte count. Pat counts 1000 red blood cells. Of these, 18 are reticulocytes. What is the percentage of reticulocytes?
-

PROBLEMS FOR MINING TECHNOLOGY PROGRAM1.5 Percents1.5.1 Percents of Numbers

- (1) If 15% of the 120 miners missed work, how many miners missed work?
- _____
- (2) The workers on 8-Main only reached 85% of their goal of 1400 tons. How many tons did the workers produce?
- _____

1.5.2 Unknown Percents

- (1) The workers on 8-Main produced 1050 tons of coal but had a goal of 1200 tons. What percent of their goal was accomplished?
- _____
- (2) Lee hauled 50 of the 153 cars of coal hauled. What percent of the total cars hauled did Lee haul?
- _____

1.5.3 Percent Reduction

- (1) The mine employed 200 workers but laid off 38. What was the percent of reduction in workpower?
- _____
- (2) The production for May was 36,000 tons and for June was 32,500 tons. What was the percent of reduction in production?
- _____

PROBLEMS FOR NURSING PROGRAM1.5 Percents1.5.1 Percents of Numbers

- (1) A child weighed 65 lbs. in January and lost 20% of body weight by April. How many pounds had been lost?

- (2) In one year, the price of vitamins has gone up by 10%. A bottle of capsules used to cost \$3.00. How much does it cost now?

1.5.2 Unknown Percents

- (1) If, on a National League for Nurses Achievement Examination in Pharmacology containing 130 questions, you answer 75 correctly, what should your score be, given as a percent?

- (2) If a person with an average daily total consumption of 3000 calories has about 450 calories for breakfast, what percentage of his total daily intake is consumed at breakfast?

1.5.3 Percent Reduction

- (1) Last year there were 232 cases of flu reported. This year there were only 198 cases reported. There were _____% less cases this year.
- (2) Hospital admissions increased 30% in 1976, decreased 20% in 1977, and increased 10% in 1978. The admissions in 1978 were what percent of admissions in 1975?

1.5.4 Change to Decimal Equivalent

- (1) State 2.12% as a decimal. _____
- (2) State 324% as a decimal. _____

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM

1.5 Percents

1.5.1 Percents and Numbers

- (1) Chris earns \$1,145 per month. Each month 7% of the salary is withheld for FICA. How much of the salary is withheld for FICA?

- (2) You estimate that 15% of your salary of \$1,145 per month is spent for food and that you save 12%.
How much do you spend for food? _____
How much do you save? _____

1.5.2 Unknown Percents

- (1) Chris's friend, Marty Fremont, is also a secretary at White Enterprises. The beginning salary was \$13,500; and at the end of the first year, the salary was increased to \$14,200. By what percent did Marty's salary increase?

- (2) Last year Chris contributed \$150 to the United Way fund drive at the office. This year Chris increased the contribution to \$175. What percent did Chris increase the contribution?

1.5.3 Percent Reduction

- (1) Bowman Office Equipment, a supplier of White Enterprises, reduced the price of a typewriter from \$496 to \$365. What percent reduction was this?

- (2) Chris learned that the price of coffee and doughnuts had fallen from \$.80 to \$.65 in the company cafeteria. What percent savings did Chris receive?

1.5.4 Change to Decimal Equivalents

- (1) The spouses of three White Enterprises executives are partners in the Golden Glow Card Shop. The profit for the last year was \$83,000 and the profits were divided as follows: Hardy, 35%; Brown 20%; and Whitman, 45%. How much did each partner receive?

Hardy _____ Brown _____ Whitman _____

BUSINESS MANAGEMENT (COND)

- (2) White Enterprises allowed discounts on the following invoice as indicated. Chris was asked by the Supervisor to calculate the amount of the discounts. What should Chris's answers have been?

<u>Amount of Invoice</u>	<u>Discount Allowed</u>	<u>Amount of Discount</u>
\$1,426.13	3 1/2%	_____
4,203.02	2 1/4%	_____
6,314.96	4 1/2%	_____

BUSINESS MANAGEMENT1.5 Percents1.5.1 Percents and Numbers

- (1) There are 780 workers at Apex Controls. The number of workers that have been employed there for over 5 years is 25% of the 780 workers. How many workers have been at Apex over 5 years?
-
- (2) If Apex had a budget of \$12,376.00 for a particular area of development, and 30% of this budget was to go towards research, then how much money is to be spent on research?
-

1.5.2 Unknown Percents

- (1) If sales for the 1984 year were \$300,000.00 and sales for the 1985 year was \$410,000.00, by what percent did sales increase in 1985 over the sales in 1984?
-
- (2) Last year we produced 6,700 heating elements, this year we produced 8,174 heating elements. What percent of production increase was there?
-

1.5.3 Percent Reduction

- (1) If you were the personnel director and you were told to cut the work force back to 663 workers from the original 780 workers. What percent of workers were cut back?
-
- (2) Apex learned that the price of wire had fallen from \$.90 ft. to \$.75 ft. What percent of savings did Apex receive?
-

1.5.4 Change to Decimal Equivalents

- (1) If a bonus of \$14,000.00 were available to be divided between 3 managers and they received the following:
 Manager A 40 % Manager B 35% Manager C 25%. How much did each manager receive?
- Manager A _____
 Manager B _____
 Manager C _____

COMPUTER TECHNOLOGY

1.5 Percents

1.5.1 Percents of Numbers

- (1) If 15% of the 140 arrays dimensioned were used, how many arrays have been used? _____
- (2) If 18% of the 500 personalized software packages each year are written for banks, then how many packages are written for banks each year? _____

1.5.2 Unknown Percents

- (1) If K.E.C. Data had a goal to produce 12 software packages in one week, but only 10 were produced, what % of their goal was accomplished? _____
- (2) Lee had coded 1000 of the 4000 lines needed for a program. What percent of coding had Lee done? _____

1.5.3 Percent Reduction

- (1) Last year floppy disk cost us \$1.20 each. This year they only cost us \$1.00 each. The floppy disk cost us _____ % less this year.
- (2) When we first purchased computer A, it had 512K Bytes of memory to use. Since then we have added some internal functions that have used up 128 K Bytes of memory, leaving us with 384 K Bytes of memory to use. What percent reduction of memory do we have? _____

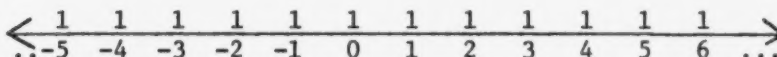
1.5.4 Change to Decimal Equivalent

- (1) If Lee had did 14.4% of a program, then how would this be wrote in decimal form? _____
- (2) What would 78% be in decimal form? _____

1.6 SIGNED NUMBERS

REVIEW NOTES

The concept of signed numbers may be most easily visualized by the aid of a number line.



Zero is in the middle with positive numbers to the right of zero and negative numbers to the left of zero. Any number to the right of another number is greater. Numbers to the left of others are less than that number.

The rules are very simple for arithmetic with signed numbers.

- 1.6.1 To add two signed numbers of the same sign, either both positive or both negative, you keep the sign you already have and add the digits. Example: $2 + 3 = 5$
 $-2 - 3 = -5$

- 1.6.2 Subtraction may be thought of as the addition of unlike signed numbers. When adding unlike signs, you keep the sign before the larger number and subtract the smaller number from the larger. Example $2 - 3 = -1$
 $-5 + 3 = -2$
 $6 - 27 = -21$
 $-1/2 + 3/5 = 1/10$

- 1.6.3 Two cases are observed when multiplying signed numbers.
 Case 1: When multiplying like signs either both positive or both negative, your answer is always positive and multiply the digits.
 Example: $(-2)(-3) = 6$
 $(2)(3) = 6$

- Case 2: When multiplying unlike signs, one positive and one negative, your answer is negative and multiply the digits.
 Example: $(-2)(3) = -6$
 $(2)(-3) = -6$

- 1.6.4 Two cases are observed when dividing signed numbers.
 Case 1: When dividing like signs either both positive or both negative, your answer is always positive and divide the digits.
 Example: $-12 \div -2 = 6$
 $12 \div 2 = 6$

- Case 2: When dividing unlike signs, one positive and one negative, your answer is negative and divide the digits.
 Example: $-12 \div 2 = -6$
 $12 \div -2 = -6$

PROBLEMS FOR MEDICAL LABORATORY TECHNICIAN PROGRAM

1.6 SIGNED NUMBERS

- 1.6 (1) Pat Precise is monitoring the equipment in the blood bank lab. Pat examines the temperature of a refrigerator. The temperature for one week read:

Sunday	Monday	Tuesday	Wednesday	Thursday
2°C	0°C	-3°C	-1°C	1°C
Friday	Saturday			
-2°C	3°C			

What was the average temperature for the week of this refrigerator

- (2) What was the difference in the temperature on Tuesday and Wednesday?
-

- (3) What was the difference in the temperature on Friday and Saturday?
-

PROBLEMS FOR MINING TECHNOLOGY PROGRAM2.1 Signed Numbers1.6.1 Addition

- (1) The profit for the Diamond #1 Mine for April, May and June was \$20,500 for April, -\$1,600 for May, and +\$28,000 for June. What was the total profit for the quarter?
-
- (2) The 5 sections at Diamond #1 Mine had the following attendance records for the week: -10, 0, -3, -8, -2. What was the total number of absences reported?
-

1.6.2 Subtraction

- (1) When Lee entered the mine Saturday morning, the temperature was 18° above zero. When Lee came out Saturday afternoon, the temperature was 13° below zero. How many degrees had the temperature dropped?
-
- (2) The elevation at the entrance to Diamond #1 Mine was 210 feet above sea level. If the elevation at the bottom of the air shaft was 208 feet below sea level, how long is the air shaft?
-

1.6.3 Multiplication

- (1) If the elevation of the bottom of a coal seam drops 2.5 feet every 100 feet of horizontal distance, how far would the elevation drop after a horizontal distance of 3200 feet?
-
- (2) How much would a mine be in debt if it were operating at a loss of \$5,000.00 per day for 30 working days if it had an initial capital of \$50,000.00?
-

1.6.4 Division

- (1) Calculate the loss per day if a company began operations with \$30,000.00 and after 25 days had a capital of -\$2,500.00?
-
- (2) If the temperature dropped from 30° above to 15° below zero in 5.5 hours, calculate the change per hour in the temperature.
-

PROBLEMS FOR NURSING PROGRAM2.1 Signed Numbers

1.6.1 Addition

- (1) PersonX gained 3 kg last month. This month, lost 5 kg.
What was the total gain or loss for the two months?
- _____
- (2) The temperature was recorded last night as -16°C . At noon,
the temperature had increased by 8° . What was the temperature
at noon?
- _____

1.6.2 Subtraction

- (1) The temperature in the freezer at 8:00 a.m. was -12°F . At
10:00 p.m. it was 3°F . How many degrees difference is there?
- _____
- (2) In an experiment, the temperature was lowered from -46°C to
 -104°C . How many degrees was the temperature lowered?
- _____

1.6.3 Multiplication

- (1) Multiply 5 by -9. _____
- (2) Multiply -7 by -4. _____

1.6.4 Division

- (1) Divide -24 by -6. _____
- (2) Divide 45 by -5. _____

PROBLEMS FOR SECRETARIAL ADMINISTRATION PROGRAM

2.1 Signed Numbers

1.6.1 Addition

- (1) Chris's supervisor interviewed three persons for a secretarial opening. Chris rated the applicants as follows: Person A, +4; Person B, -2; and Person C, +1. What was the total of all the ratings added together?
- _____

- (2) Persons who were interviewed for an accounting position were rated as follows: Person A, +3; Person B, -4; Person C, -3; Person D, +6; Person E, -1; What was the total of all ratings?
- _____

1.6.2 Subtraction

- (1) The temperature dropped from 20° to -5°. How many degrees did the temperature fall?
- _____
- (2) The thermometer outside Lee's office window fell from 14° to -8°. How many degrees did the temperature fall?
- _____

1.6.3 Multiplication

- (1) Chris kept a friend Friday night while the parents attended the theater. Chris was asked to help with the following problems. Calculate the answers.

$$\begin{array}{r} -3 \\ \times 4 \\ \hline \end{array} \quad 2 \times -8 = \underline{\hspace{2cm}} \quad \begin{array}{r} -2 \\ \times -20 \\ \hline \end{array} \quad \begin{array}{r} -3.61 \\ \times 2.8 \\ \hline \end{array}$$

1.6.4 Division

- (1) Chris also helped the friend with the following problems. See if you can work the problems.

$$10 \overline{) -40} \quad -60 \div -5 = \underline{\hspace{2cm}} \quad -20 \div -2 = \underline{\hspace{2cm}}$$

$$\begin{array}{r} -15 \overline{) 75} \end{array}$$

PROBLEMS FOR BUSINESS MANAGEMENT1.6 Signed Numbers

1.6.1 Addition

- (1) The four quarterly profits for Apex Controls this year were:

quarter 1	\$ 9,867.42
quarter 2	4,122.17
quarter 3	1,785.96
quarter 4	44,904.09

What was the total profit for the year? _____

- (2) Sales last week were up by \$4000.00. Sales this week were down by \$6000.00. What was the total gain or loss for the two weeks? _____

1.6.2 Subtraction

- (1) Apex uses a temperature control unit for some materials that must be kept cool until they are ready for use. This morning when the temperature was checked the reading was -18°F . At the end of the shift the reading was 9°F . How many degrees difference is there? _____
- (2) The heater thermostate that we make have a temperature range from -10°F to 98.7°F . How many degrees difference is there? _____

1.6.3 Multiplication

- (1) If a company lost \$2,547.16 a week for three weeks, how much money did it loose? _____
- (2) Work the following problems:

$$-6 \times -9 = \underline{\hspace{2cm}}$$

$$4 \times -6.8 = \underline{\hspace{2cm}}$$

$$-20 \times -30 = \underline{\hspace{2cm}}$$

1.6.4 Division

- (1) Divide -42 by $-7 = \underline{\hspace{2cm}}$
- (2) Divide 81 by $-9 = \underline{\hspace{2cm}}$

PROBLEMS FOR COMPUTER TECHNOLOGY1.6 Signed Numbers

1.6.1 Addition

- (1) In a particular program two fields were to be added together when the user entered the data. Below are some of the data entered. Tell the results.

- A. $7 + -14$
 B. $-320 + -46$
 C. $\$21.17 + -\$4.90$
 D. $-33,200 + 28,641$
 E. $-42,031 + -16,817$

Results

1.6.2 Subtraction

- (1) One of the programs written was for a weather forecasting application. Below are the morning and evening temperatures of given days. Tell how many degrees the temperature dropped for each day.

	<u>Morning</u>	<u>Evening</u>
Monday	14°F	-2°F
Tuesday	3°F	-8°F
Wednesday	43°F	-1°F
Thursday	22°F	-18°F
Friday	17°	-9.8°

1.6.3 Multiplication

- (1) In a program for electronics, the amps is multiplied by the resistance to give the voltage. Determine the voltages in the following.

<u>Amps</u>	<u>Resistance</u>	<u>Voltage</u>
-3	24	_____
-6.8	2	_____
-27.6	14.8	_____

- (2) Do the following multiplication problems:

- A. $-3 \times -3 =$ _____
 B. $-24 \times -4.2 =$ _____
 C. $18 \times -5 =$ _____
 D. $-236 \times -3.2 =$ _____

1.6.4 Division

- (1) Divide -108 by 9 = _____
 (2) Divide 180 by -15 = _____

2.0 Reference Section: Algebra

21.1. Algebraic Expressions

Algebra is based on the same concepts of numbers as arithmetic. However, in arithmetic, usually the only unknown is the answer. In algebra, we often know the answer but do not know the variable that produced the answer. A letter of the alphabet (X or N or some other letter) is used in place of a number. Since these letters can take on different values, we call them "variables." Remember, these letters (variables) are representing real numbers. When we "solve" an algebraic expression, we are simply looking for those values of the variable which make the equation true.

2.1.2 Equations

Equations are simply sentences expressed with mathematical symbols. For example, two plus seven is nine ($2+7 = 9$). This is a closed sentence. An open sentence has a place for something to vary; some number plus 3 is 12. Not just any number (N) will work. We observe the N must be nine or $N = 9$.

Notice an equation is like a see-saw. What is done to one side must be done to the other to keep things balanced. In $N + 3 - 3 = 12 - 3$ or $N = 9$.

Likewise, suppose $3N - 2 = 7$. This is a little more complicated. First we want N or multiples thereof to be alone. Start by adding '2' to each side of this equation.

$$3N - 2 = 7$$

$$3N - 2 + 2 = 7 + 2$$

$$3N = 9$$

Now we see we must divide both sides by 3 to get N alone.

$$\frac{3N}{3} = \frac{9}{3} \quad \boxed{N = 3}$$

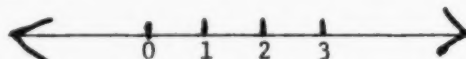
Algorithms for Equations

- A. Express English sentence to mathematical equation symbol form.
- B. Add or subtract on both sides (additive inverse) to get rid of unwanted numbers.
- C. Multiply both sides to get rid of unwanted multiples.

2.1.3 Inequalities

Inequalities deal with expression which are not equal to each other; therefore, values are less than ($<$) or greater than ($>$) one another. For example $2 < 3$, $5 > 1$, $-2 < -1$. Really any number that is to the right on a number line of another number is defined as greater than.

In inequations, $X > 3$, say for example, means any number to the right of 3 on a number line.



Inequations are solved like equations with the exception that when multiplying or dividing by a negative, always change the direction of the inequality sign. Example: $-2X < 4$. To get X by itself, we must divide both sides by -2 . So $-2X < 4$

$$\frac{-2X}{-2} > \frac{4}{-2}$$

Now, change the direction of the sign to "get rid of" negatives.

$$\underline{X > -2}$$

So any number to the right of -2 will solve the inequation.
TRY IT.

Algorithms for Inequalities

- A. Express English sentence to mathematical equation symbol form.
- B. Add or subtract on both sides (additive inverse) to get rid of unwanted numbers.
- C. Multiply both sides to get rid of unwanted multiples.

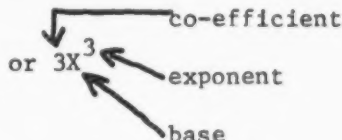
2.2 EXPONENTS IN ALGEBRA

An exponent is a small number written to the right and above the expression that is being raised to a power. This means to multiply the expression by itself the indicated number of times. For example, 2^3 means $2 \times 2 \times 2$ or 8.

Another example $(2X)^3$ means $(2X)(2X)(2X)$ or $8X^3$. Note that $2X^3$ does not mean $(2X)^3$ or $(2X) \times (2X) \times (2X)$ or $8X^3$. Remember $2X^3$ means $2 \times X \times X \times X$. The exponent refers only to the number of variable it follows. Parentheses must be used to set off more than one character if that is desired.

Some terms will be helpful to keep this straight. 'Base' refers to the expression modified and exponent refers to the modifier. These take the general form:

(co-efficient)(base)^{exponent} or $3X^3$



(Example)

2.2.1 A term in an algebraic expression means a 'bundle', so to speak without being attached by plus or minus signs to other terms. Often we call these terms monomials for one unit number bundle. Some examples of monomials are: 2, -5, $3X$, $6X^2$, $-5X^3$, $10X^4Y^5$, etc. A monomial then is any expression made up of the form aX^q where a is any real number and q is any rational number. This would include all monomials in X .

2.2.2 To add or subtract monomials, the bases and the exponents must be exactly the same. The coefficients, that is, the numbers out front, may be different. It is similar to counting the contents in a fruit basket. Each monomial is like a certain kind of fruit. One could add only those like fruit in order to get a meaningful summation. For example, one apple + 2 oranges + 4 bananas + 1 apple + 1 orange + 1 banana equals 2 apples, 3 oranges, 5 bananas.

Given an algebraic expression, the algorithm for the addition of terms is as follows:

- A. Note those terms that are alike both in bases and exponents.
- B. Arithmetically add the coefficients of like terms.
- C. Write the total of the coefficients of like terms and copy the bases and exponents like they are.

Example: Add like terms in:

$$5X^2 + 3XY + 2X^2 + 6XY$$

Note circles and squares will be placed around the like terms. Remember to include the sign in front along with the coefficient.

$$\textcircled{5X^2} + \boxed{3XY} + \textcircled{2X^2} + \boxed{6XY}$$

Now add coefficients.

$$\underline{7X^2 + 9XY}$$

Another example: Add and simplify:

$$\textcircled{5XY^2} + \textcircled{6X^2Y} + \textcircled{2XY^2} + \triangle 5X + \boxed{7X^2Y} + \triangle 2X$$

Suggestion: Draw like geometric shapes around like monomials and then just add the numbers in front.

$$7XY^2 + 13X^2Y + 7X$$

2.2.3 Subtraction of Terms

Subtraction remember is nothing but the addition of negative numbers. Therefore, the algorithm is exactly the same for subtraction of terms as adding of terms. Simply keep the rules for addition of signed numbers when adding coefficients.

For example: simplify:

$$5AB + 3XY - XY + 2AB - 5XY - 4AB$$

Again, draw geometric shapes around like terms, including the plus or minus sign in front of the coefficient.

$$\textcircled{5AB} + \boxed{3XY} - \boxed{XY} + \textcircled{2AB} - \boxed{5XY} - \textcircled{4AB}$$

Now, add the signed coefficients to get:

$$\underline{3AB - 3XY}$$

2.2.4 Multiplication of Terms

The algorithm for multiplying terms is as follows:

- A. Copy the base or bases of the monomials.
- B. Add exponents (by rules of signed numbers) to like bases.
- C. Multiply coefficients by rules of signed numbers.

Now you can multiply or divide any two things together. However, remember you cannot add or subtract unlike monomials.

Example applying the above algorithm:

Multiply: $(3X^4Y^2)(-2X^2Y^3)$

- A. Copy bases $\longrightarrow$ $X \ Y$
 B. Add exponents $\longrightarrow$ $X^6 \ Y^5$
 C. Multiply coefficients $\longrightarrow$ $-6X^6 Y^5$
Answer: $-6X^6 Y^5$

2.2.5 The algorithm for Division of Terms is as follows:

- A. Copy bases.
 B. Subtract the exponent in the denominator or the one you are dividing by from the upper one.
 C. Divide coefficients by rules for signed numbers.

Example: $\frac{12 W^3 X^4 Y^2}{-4 X^2 Y^3}$

- A. Copy bases $\longrightarrow$ $W \ X \ Y$
 B. Subtract exponents $\longrightarrow$ $W^{3-1} \ X^{4-2} \ Y^{2-3}$
 this becomes $W^2 \ X^2 \ Y^{-1}$
 C. Divide coefficient $\longrightarrow$ $\frac{12}{-2} = -6 \longrightarrow -6 W^2 X^2 Y^{-1}$

Now if you end up with a negative exponent, you may make it positive and place the base and its exponent in the denominator. Our answer becomes:

$$\frac{-6W^2 X^2}{Y}$$

2.2.6 Terms raised to powers follow a simple algorithm.

- A. Treat each base and coefficient as a base and raise each of them to the indicated power.
 B. Multiple exponents.

Examples: $(-2X^2Y^3)^2 = (-2)^2 (X^2)^2 (Y^3)^2$

Another example: $(3X^{-2}Y^4)^{1/2} = (3^{1/2}) (X^{-2})^{1/2} (Y^4)^{1/2} =$

$$\sqrt{3} X^{-1} Y^2 \quad \text{or} \quad \sqrt{\frac{3 Y^2}{X}}$$

So raising powers to powers we simply multiply exponents.

2.3 PROPORTIONS IN ALGEBRA

2.3.1 Proportions may be thought of as comparables or an equivalent quantity as compared to the same ratio of quantities in a previous comparison. Suppose for example that I have eight rabbits and three are white and five are black. Also, suppose that I am going to buy 32 more rabbits and I want to keep the same proportion of white to blacks. How many must be white and how many black? Let X be the number of whites then $32-X$ represents the blacks. We will set up an equation for comparison white to black and white to black after new purchase in fractional form: $\frac{3}{5} = \frac{X}{32-X}$

To solve any fractional equation we cross multiply $5X = 3(32-X)$ or $5X = 96 - 3X$ $8X = 96$ or $X = 12$ so we shall buy 12 white rabbits and $32-12=20$ black rabbits.

2.3.2 Mixtures or blends are very important especially in the coal fields when desiring certain ash, moisture, BTU content for each ton of coal. Many times different seam qualities must be mixed in order to obtain the proper specifications of a coal buyer. Suppose a utility company must have exactly a 10% ash coal for their furnaces to operate properly. Now suppose your Darby # 5 Seam of coal is 7% ash per sample pound. Also suppose your Splint #12 seam is 12% ash per sample pound. How many pounds of the #5 seam; must you mix with the #12 seam to get exactly a 10% ash blend?

We can approach this problem on a ton basis. Let X be the number of pounds of #5 seam; therefore, $2000-X$ is the number of pounds of # 12 seam because there are 2000 pounds in a ton. Now if we apply the percentages of ash before and after blending we see that is equal. In other words:

$$.07(X) = .12(2000-X) = .1(2000)$$

This is the approach that is always used in setting up a working equation to a real life (word) problem. You always examine to see what is equal before and after some problem interaction.

$$\text{Again } .07(X) + .12(2000-X) = .1(2000)$$

$$.07X + 240 - .12X = 200$$

$$.07X - .12X = 200 - 240$$

$$-.05X = -40$$

$$X = \frac{-40}{-.05}$$

$$X = \frac{40}{.05}$$

$$X = \frac{4000}{5}$$

$$X = 800$$

So X relates to the #5 Darby Seam of 800 pounds and $2000 - X = 1200$ pounds of the Spling #12 Seam.

Thus 800 pound of #5 at 7% plus 1200 pounds of #12 at 12% yields 2000 pounds of Blend at 10%.

- 2.3.3 There are a couple of observations that make mixture and proportion type problems more easily mastered. Always remember that when setting up an equation we are looking for something to be equal on both sides. Let us suppose after a cold Kentucky winter that Joe Student needs to increase the percentage of anti-freeze coolant in the 1939 Ford Coupe. How much pure (that is 100%) anti-freeze must be mixed with the present 8 quarts of 20% solution to make a 35% solution?

- A. Technique Always draw a picture if at all feasible.

8 quarts 20% afz	+	X quarts 100% afz	=	8 + X 35% afz
------------------------	---	-------------------------	---	---------------------

See now that the amount of anti-freeze (afz) on the left sided containers equals the amount in the right hand container.

- B. Thus we have an equation.

- C. Simplify the equation and isolate the unknown.

$$20\% (8) + 100\% (X) = 35\% (8+X)$$

$$.2 (8) + 1. (X) = .35 (8+X)$$

$$1.6 + X = 2.80 + .35X$$

$$X - .35X = 2.8 - 1.6$$

$$X = \frac{1.2}{.65}$$

$$X = 1.85 \text{ quarts}$$

So we need to add 1.85 quarts of pure anti-freeze to make a 35% solution having a total volume of 9.85 quarts.

However, the Student has a problem. The radiator is filled to the neck, and cannot add anything more or else it will spill. How much of the 20% solution must be taken out and how much of the 100% pure anti-freeze must be added to make a 35% solution in the 8 quart radiator?

- A. Again, draw a picture.
- B. Set up an equation by finding equals. Here we really want to know how much to drain out because an exact amount of pure anti-freeze will be added to make a 35% solution

$$\boxed{\begin{array}{c} 8 - X \\ 20\% \end{array}} + \boxed{\begin{array}{c} X \\ 100\% \end{array}} = \boxed{\begin{array}{c} 8 \\ 35\% \end{array}}$$

so

$$20\% (8-X) + 100\% (X) = 35\% (8)$$

$$.2 (8-X) + X = .35 (8)$$

$$1.6 - .2X + X = 2.80$$

$$.8X = 2.8 - 1.6$$

$$.8X = 1.2$$

$$X = \frac{1.2}{.8}$$

$$X = 1.5$$

So Pat must drain out 1.5 quarts and replace it with 100% pure anti-freeze to get the needed 35% solution.

As you have undoubtedly noticed, this last section has dealt with far more demanding concepts. Throughout our manual, we have been leading toward this type of complex Problem Solving. The last section on Algebra (2.5) begins to look at the process as one of translation. The student must master the art of translating English sentences containing numbers into number sentences. Tables will be provided to assist you in this process. If you have mastered the concepts in Section 1.1 through 2.4, you already know enough math to be a good problem solver. What you have to do now is "bridge the gap" between word problems and number sentences. After that, you should be able to refer to the mathematical "mechanics" in Section 1.1 through 2.4.

Additional reference sections are provided for more advanced review on Concepts (3.0) and Applications (4.0). These should not be attempted until you have mastered 1.0 (Arithmetic Skills) and 2.0 (Algebraic Skills).

2.4 NUMBER SENTENCES

REVIEW NOTES

Perhaps the best known (and most feared) applications of algebra lies in the solution of "word problems". The mathematical operations of algebra are basically solved by almost mechanical operations, and we have seen that these may usually be completed with a finite series of steps, arranged in an algorithm. Also, many students are able to avoid an algebraic solution of "word problems" by applying their knowledge of arithmetic in a trial and error fashion. However, this takes a lot longer and often leads to a bewildering series of attempted calculations in order to solve a rather simple problem.

In this final section on Number Sentences, we will present a systematic way to approach word problems. Not only will you be able to solve more types of problems - you will save time on problems which had previously been solved through "trial and error".

Compare the two equations below:

Word Problem

What number increased by 3
is equal to 10?

Algebraic Expression

$$A + 3 = 10$$

In this simple example, we can see that the two expressions are saying the same thing. We could substitute numbers for the word problem until the answer is found. Or, we could solve the problem by the rules of algebra which we have already learned:

Algebraic Expression

$$\begin{array}{rcl} X + 3 & = & 10 \\ - 3 & = & -3 \\ \hline X & = & 7 \end{array} \quad \begin{array}{l} \text{(Subtracting} \\ \text{3 from both} \\ \text{sides)} \end{array}$$

If we add more words, the problem appears harder:

If Pat's age is increase by three, Pat would be as old as Chris. Chris is 10. How old is Pat?

Fortunately, the algebraic expression is still $X + 3 = 10$, so we still have a rather simple problem. However, a new skill is becoming important - that of TRANSLATING the word problem into an algebraic expression. This is the main feature that troubles so many students when they attempt to solve word problems.

The Table on the next page gives most of the expressions found in word problems and their equivalent expression in algebraic terms. Study that table until you are confident that you understand the "vocabulary" of word problems. Then study the STEPS FOR SOLVING WORD PROBLEMS. Finally, you should work through the examples. We believe that you will find this a helpful approach for applying what you already know about algebra. More importantly, it will provide a systematic way to approach word problems.

2.4.1 TABLE FOR TRANSLATING WORD PROBLEMS TO ALGEBRAIC EXPRESSIONS

Word Problem FormatAlgebraic EquivalentAddition

3 more than a number.....	$X + 3$
a number increased by 4.....	$X + 4$
the sum of A and B.....	$A + B$
a number added to 6.....	$X + 6$

Subtraction

4 less than a number.....	$X - 4$
A decreased by 7.....	$A - 7$
The difference between A and B.....	$A - B$

Multiplication

A multiplied by B.....	$A \cdot B$
The product of A and B.....	$A \cdot B$ (or $B \cdot A$)
Three times the number.....	$3 \cdot X$ (or $3X$)
$\frac{3}{4}$ of a number.....	$\frac{3}{4} \cdot X$
A times B.....	$A \cdot B$
27% of a number	$0.27 \cdot X$

Division

A divided by B.....	$\frac{A}{B}$ (Same as $B \overline{)A}$)
The ratio of A and B.....	$\frac{A}{B}$ (Note that the variable or number is the numerator.)
The quotient of A and B.....	$\frac{A}{B}$ (Note as above)

Division often puzzles problem solvers because they do not know which number to divide by. Unless there is other information, assume that the first mentioned number will be in the numerator (on top) of the ratio. That is, the first named number or variable is usually the numerator.

2.4.2 STEPS FOR SOLVING WORD PROBLEMS

- A) Read and reread the problem carefully.
- B) If possible, draw a diagram to help clarify the relationships.
- C) Assign a letter for the unknown number (usually X, although any letter will do, of course).
- D) Translate the words into algebraic information using all of the information from Steps B and C. Note that sometimes problem writers will include unnecessary information. Diagrams (Step B) help eliminate these "traps" better than anything else.

STEPS FOR SOLVING WORD PROBLEMS (COND)

- E) Write an equation from Step D.
- F) Simplify the equation, combining like terms and reducing expressions.
- G) Isolate the unknown (your assigned letter) on one side of the equal sign.
- H) Check the solution by substituting into your equation.

2.4.3 Applying The Steps

Example Chris is two years younger than Marty and the sum of their ages is 32. How old is each?

- A) Read and Reread - on your own.
- B) Draw a diagram - a little difficult here, unless you try the number line:

```

-----*****
      0      10      20      30      32      ----Chris
      *****Marty
  
```

Notice that we are not trying to solve the problem with this approach. The diagram just helps us get a "feel" for the numbers that might work. Chris's line (----) is shorter; Marty's line is longer. Both lines together equal 32. These are the main aspects of the problem. Surely we will not make a careless error and decide that Chris is five years old. Our diagram keeps us closer to the accurate answer than that.

- C) Assign a letter for the unknown number. Really, we are looking at two related numbers, so we could assign "X" to either Chris or Marty. We'll pick on Marty for our example:

$$\text{Marty's Age} = X$$

- D) Translate the words into algebraic information. Here is the main part of the problem. Let's see what we know and use the table to translate that into algebraic terms:

<u>What we know</u>	<u>Equation</u>
Marty's age is X.	
Chris's age is two years younger than Marty.	Chris = X - 2

- E) Write an Equation. Look for the key relationship and try to translate that part into algebraic terms:

<u>What we know</u>	<u>Equation</u>
The sum of Marty's and Chris's ages is equal to 32.	$X + (X - 2) = 32$

Applying the Steps (Cond)

- F) Simplify the equation. We're back to simple algebra now, so the rest should be easy. Combining the like terms:

$$X + (X - 2) = 32 \text{ will give us}$$

$$2X - 2 = 32$$

- G) Isolate the unknown on one side of the Equal Sign. We can do this here by adding two to the left side of the equation, leaving $2X$ isolated. Remember, however, if you have done something to one side of the equation, you must do exactly the same thing to the other side or the equation will be "out of balance".

$$2X - 2 = 32$$

$$+ 2 = +2$$

$$2X = 34$$

Therefore $X = 17$ (Marty's age)

- H) Check the solution by substitution:

$$17 + (17 - 2) = 32. \text{ Of course, Chris is 15.}$$

3.1 MEASUREMENT DEFINITIONS

Measurements are simply concepts that have been quantified numerically. Most things in our universe could not be discussed without measurements. Measurement involves many observable concepts such as height, weight, temperature, pressures, intensity levels for sound and light, distances, angles, areas, volumes, and the list goes on. In all of these some sort of numerical comparison, a number scaled comparison is used to convey the message.

3.1.1 Length

Length may be thought of as linear distance along a line or line segment. The measure of our height is a good example. Many standards of unit comparison are used. One familiar to you is the English system of Linear Measurement with its inches, feet, yards, miles, etc. These may be used for linear definitions or you may create some units of your own. Perhaps you could create a unit called "my-thumb" for measuring lengths. The length of a pencil is about $2 \frac{1}{2}$ "my thumbs" long. Notice how numbers must be used with particular units to describe length.

MEASUREMENT DEFINITIONS (COND)

3.1.2 Perimeter

Perimeter is the distance around some outside edge of an object or geometrical shape. The perimeter of Southeast Community College is several miles. The perimeter of most coffee tables would be about 10 feet. When perimeter is measured for circular objects we call that perimeter a circumference. The circumference of the earth is approximately 25,000 miles.

3.1.3 Mass

Mass is the concept concerned with quantity of material in relation to its weight. Mass is measured in units such as ounces, pounds, tons, grams, kilograms, etc. The heavier an object is, the more massive it is and the more force it takes to move it. For example, a locomotive is more massive than a bicycle.

3.1.4 Area

Area can be simplified to be the concept of unit squares over some surface. These squares may be any unit size. Standard units of squares are inches, yards, centimeters, meters, miles, etc. For example, a classroom at SECC may be about 600 square feet; which means 600 square, each one foot by one foot, could be placed on the floor in the room.

3.1.5 Volume

Volume is simply the concept of space contained in something. Volumes contain amounts. Volume can be thought of as areas stacked on top of one another creating a third dimension. Standard units for volumes are cups, pints, gallons, liters, etc.

3.1.6 Angles

Angles involve the concept of turning, where one complete turn or revolution is the basic concept. The standard unit most often used is $1/360$ of a whole revolution called a degree. If something is turned 30 degrees, it has been turned $1/12$ of a whole turn.

3.2 NUMBER FORMS

The previous section dealt with Measurement (3.1) of various shapes. We saw how numbers become a "common language" for measuring and comparing objects which are very different. In this section, we will review the different formats used for numbers for example, there are specific rules for rounding off numbers, for expressing them as logarithms, as roots, etc. Finally, we will study the manner in which numbers are expressed in scientific notation. This last concept has become particularly important since computer usage has become widespread in the vocational fields.

3.2.1 Rounding Off Numbers

When we divide numbers or take square roots of numbers, we often come up with answers that have many additional places to the right of the decimal. For example: $.1/3 = 0.3333333333$, etc. Even with calculators and computers available, it is convenient to eliminate these small fractions so that we can combine different numbers. Consider how awkward it is to add 0.333333 plus 1.25 plus 2.5 plus 3.1416.

Usually we decide how accurate we want to be. For example, we may want our results to be accurate to the hundredth place. That would mean it is accurate to the second digit to the right of the decimal. (The student may want to review the various place values, explained in Section 1.3)

Algorithm for Rounding Numbers

Rounding is usually done to the right of the decimal point. Example: Round 32.4536 to the thousandth place.

- A. Algorithm: Multiply number by inverse of place value to be rounded. In our example, that is 1000.
- B. Add 0.5 to this number (from A) becoming 32454.1.
- C. Throw away any digits to the right of the decimal becoming 32454.
- D. Now divide the number by 1000 becoming the proper rounded number to the thousandth place of 32.454.

3.2.2 Roots

Another number form which is frequently encountered is that for roots of a number. All of us are familiar with square roots, but it is often convenient to deal with higher order roots.

Given n B means what number n can be multiplied by itself n times to give B. Examples are square root, cube root, fourth root, etc.

$$\sqrt[2]{100} = 10$$

That is, 10 multiplied by itself twice yields 100.

$$\sqrt[3]{8} = 2$$

2 multiplied by itself three times is $2 \times 2 \times 2 = 8$.

$$\sqrt[4]{81} = 3$$

$$\sqrt[5]{32} = 2$$

$$\sqrt[n]{1} = 1$$

Thus, roots may be viewed as a symbolic way of expressing multiplication.

3.2.3 Scientific Notation

This is a useful shorthand for expressing either very large or very small numbers which occur in scientific and technical applications. Consider how much easier it is to use scientific notation for a very large number:

<u>Standard Notation</u>	<u>Equivalent in Scientific Notation</u>
13,000,000,000,000	13×10^{12}

You may have already figured out that the process is based on exponents and/or roots. The word exponent (see Section 2.3 for more details) simply means repeated multiplication. Thus, 2^3

means $2 \times 2 \times 2 = 8$

8^{-3}

means $\frac{1}{8 \times 8 \times 8}$ or $\frac{1}{512}$

one million means $1,000,000$ or 10^6

The following algorithm will reduce the process of translating standard notation into scientific notation into a few simple steps.

Example: Express 328 million dollars (\$328,000,000) in the shorthand scientific notation.

Algorithm: To put any number in standard notation:

- Move the decimal to the right or the left until only one nonzero digit remains to the left of the decimal.

- B. If you moved to the left n times your exponent will be positive n or just n .
- C. If you moved the decimal to the right n places, your exponent will be negative n ($-n$). So 328000000 is 3.28×10^8 because we moved to the left 8 times.

Likewise, .0000328 is 3.28×10^{-5} because we moved 5 places to the right.

$$\text{Remember } 10^{-n} = \frac{1}{10^n}$$

or

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$$

3,2,4 Logarithms

Logarithms are number forms based on exponents and roots. "Logs" were developed prior to the computer age to facilitate computation of very large and very small numbers. They still have many applications and have been used in all vocational fields. The student should study the following section to understand what they are and how they are computed. The specific applications you may encounter are far too diverse to study in this brief manual.

Example: $\log_b N = e$ where e is the logarithm of N to base b .

In exponential notation, this means $b^e = N$. Most of the time we speak of common logs or logs to the base ten.

$$\text{Example: } \log_{10} 100 = \underline{\quad ? \quad}$$

$$\log_{10} 100 = x$$

$$\text{means } 10^x = 100$$

Of course, $x = 2$, so 2 is the log of 100.

Example: Find log of 238000 means what exponent ten must be raised to give this

$$10^x = 238000$$

5

So now we know the log of 10 , that is 5 and if we add the log of 2.38 to 5 we are finished. Logically, the log of 2.38 is between 0 and 1 because $10^0 = 1$ and $10^1 = 10$.

No person remembers these values of decimal numbers so we look up the value in a Table of Logarithms.

So ten to what power is 2.38. Answer: .376577.

So log 238000 is 5.376577 or if you want to round off to four places by the procedure of 2.1.4 the final answer is 5.3766.

If you want to find the number which is associated with a particular logarithm, you have to look up the log value in a table, find the number associated with the value in the table (usually in a column of numbers to the left) and go through the translation from scientific notation to standard notation in reverse.

This reversed procedure is used to find the analog when you have started with the logarithm. Almost all analog problems assume the base 10 since almost all of the logarithm tables are used on 10 as a base.

We hope this section will help you understand what logs and analogs are. From a practical standpoint, everyone uses computers to find log values today. The log function is almost always available on the simplest microcomputer and even on many hand calculators. Study the instruction manual for whatever calculator or computer you are using to determine the specific directions for finding logs. However, you will need to know the vocabulary and concepts in this section to understand what is going on.

3.3 NUMBER THEORY

REVIEW NOTES

This section contains some important vocabulary and concepts for beginning more advanced mathematics. Many of these do not have immediate application in the vocations. However, all of these are important for the well-rounded education of the vocational student.

3.3.1 Number System

A number system must include a Set of Elements (e.g. numbers) and one or more Operations (e.g., addition, subtraction, etc.) for combining those elements. For example, we have discussed the Whole Number System (Section 1.1). This system has whole numbers as elements and a variety of operations.

3.3.2 Open and Closed Operations

An operation such as division is considered closed if the result of the operation is an element with the number system. For example: $8 \div 4 = 2$. This operation is closed within the system of whole numbers since the result (2) is an element of that system.

Any operation which is not closed is described as Open. That is, the result is not an element of that number system. For example: $4 \div 3 = 1.33$. The result (1.33) is not a member of the Whole Number System, so we can say that the operation of division is open for the whole number system.

3.3.3 Identity Element

The identity element for a given operation (e.g., addition, subtraction, etc.) is an element of the number system which does not change other elements of the system for that operation. For example:

Operation	Identity Element	Example
Addition	0	$3 + 0 = 3$
Subtraction	0	$4 - 0 = 4$
Multiplication	1	$3 \times 1 = 3$
Division	1	$3 \div 1 = 3$

3.3.4 Inverse Element

The inverse element for a given operation is an element of the number system which yields a result of zero (0) for an element. Thus, an element such as +1 has an inverse element (-1) for algebraic addition.
 $(+1) = (-1) = 0$

3.3.5 Parenthesis

Students often do not recognize the importance or the simplicity of parenthesis. In mathematical systems, parenthesis is order to do that operation first. If there are several parenthesis, you start with the innermost parenthesis. For example, to simplify the following expression, just start with the innermost parenthesis, doing that operation first.

$$((3(2 + 5)) + 4)$$

innermost parenthesis

$$((3(7)) + 4)$$

next innermost

$$(21 + 4)$$

$$25$$

NOTE that the parenthesis means to multiply if there is no other symbol such as + or -.

Thus, parenthesis specify the order in which you complete mathematical operations (within a complex expression). This is true for arithmetic operations, algebraic operations, statistics, and even statements on number theory.

3.3.6 Associative Properties

Mathematical operations are described as "associative" if parenthesis do not affect the result. For example, the operation of addition is associative:

$$(6 + 3) + 2 \quad \downarrow \quad 9 + (3 + 2)$$

$$\downarrow$$

$$9 + 2$$

$$11$$

$$\downarrow \quad 6 + 5$$

$$\downarrow \quad 11$$

Since the parenthesis do not affect the results.

However, subtraction is not associative.

$$(6 - 3) - 2 \quad \downarrow \quad 6 - (3 - 2)$$

$$\downarrow$$

$$3 - 2$$

$$1$$

$$= 6 - 1$$

$$= 5$$

3.3.7 Commutative Properties

Mathematical operations are described as commutative if the order of the elements in an expression does not affect the result. For example, addition and multiplication are commutative:

$$6 + 3 = 3 + 6$$

$$6 \times 3 = 3 \times 6$$

However, subtraction and division are not commutative:

$$6 - 3 \neq 3 - 6$$

$$6 \div 3 \neq 3 \div 6$$

3.3.8 Types of Numbers

There are many types of numbers. Learning the vocabulary to distinguish between these types is an important step toward continuing your mathematics improvement.

Natural Numbers or Counting Numbers are those numbers which were immediately obvious when human beings started counting things. For example, the following are natural numbers: 1, 2, 3, 4, 5, 6, ..., 889, ... 901, etc. Note that zero is not included. Human beings did not see much of a need to count "nothing" at first. In fact, many centuries passed before the importance of zero was understood.

Prime Numbers and Composite Numbers: Among the natural numbers are some that can be evenly divided only by the number 1. These are called "prime numbers." (Note that the word "prime" means "first" or "the number one choice.") Examples of prime numbers include 2, 3, 5, 7, 11, 13, etc.

All of the missing numbers in this sequence can be evenly divided by natural numbers other than one so they are not prime numbers. Note that "two" is the only even prime number of all numbers.

All of the missing numbers (except one itself) in this sequence are called composite numbers because they are composed of (the product of) numbers other than 1. For example:

$$4 \text{ can be composed of } 2 \times 2$$

$$6 \text{ can be composed of } 3 \times 2$$

$$8 \text{ can be composed of } 4 \times 2$$

In fact, all composite numbers can be composed of only one set of prime numbers. We are assuming that the order in which they are multiplied does not matter. That is, $4 \times 2 = 2 \times 4$ from the commutative property of multiplication. You may think, for example, that 20 can be composed of several pairs of numbers. For example:

$$1 \times 20 = 20$$

$$2 \times 10 = 20$$

$$4 \times 5 = 20$$

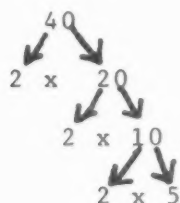
However, none of these combinations can be considered since they all involve numbers that are not primes (1, 10, and 4). Only one combination will work: $2 \times 2 \times 5 = 20$. The fact that this is true (all composite numbers can be the product of only one set of prime numbers) is considered the Fundamental Theorem of Arithmetic.

Factors: Interestingly, no systematic procedure (other than trial and error) is known to evaluate whether a certain large number (e.g., 1211) is a prime number. Of course, we know that 2 is the only even prime number, but it gets more difficult after we have eliminated all even numbers.

There is, however, a simple and systematic way to determine the factors of a composite number. We repeatedly divide the composite number by the lowest prime number which will yield a whole number as the result. For example, what are the factors of 30? Just keep dividing until nothing but prime numbers are left.



The factors are the prime numbers which result from this repeated division (here, $2 \times 3 \times 5 = 30$). Again, find the factors for 40.



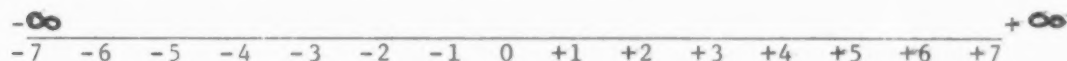
The factors of 40 are $2 \times 2 \times 2 \times 5$.

A final example where a different lowest prime must be used is 27.



The factors of 27 are $3 \times 3 \times 3$.

Integers: The system we refer to as "integers" includes the counting (natural) numbers, zero, and the inverse of the counting numbers. Remember that inverse of +3 is -3; of +9 is -9, and so on. Thus, you may recognize the integers as all of those numbers we have entered on the 'number line':



Rational Numbers: We have other types of numbers to consider, of course. Consider the fractions which are not included on the number line for integers. A number such as $+1\frac{1}{2}$ would fall

between +1 and +2. All other possible fractions ($1\frac{3}{4}$, $1\frac{7}{8}$, $1\frac{2}{31}$,

etc.) are also classified within the Rational Number System.

More generally, the rational numbers include all numbers which can be expressed in the form A/B where A and B are both integers. Note that this includes all possible positive integers, all possible positive fractions, all possible positive compound numbers (e.g., $1\frac{1}{2}$, $2\frac{3}{4}$, etc.), the number zero, and all negative inverses of these numbers. Do not forget that 1 and 2, and 33, etc. are both integers and rational numbers. They satisfy the A/B form since all of them can be divided by 1.

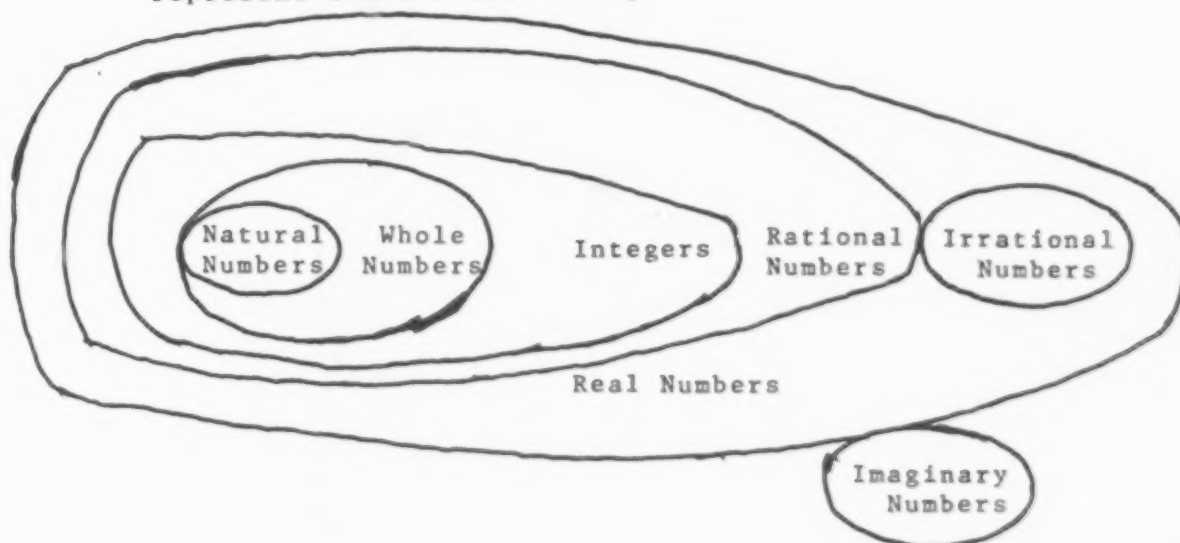
Another kind of rational number is found in the nonterminating decimal number. For example, when we divide out the fractions $1/3$ or $2/3$, we could go on indefinitely with the division. That is: $1/3 = 0.333333333333...etc.$
 $2/3 = 0.666666666666...etc.$

Interestingly, these nonterminating decimal numbers cannot ever be precisely stated in decimal terms. Still, we can rationally understand these numbers precisely as fractions (e.g., $1/3$ or $1/6$).

Irrational Numbers: Nonterminating decimals that do not repeat the sequence are called irrational numbers. These are easy to construct: 3.2140693853 etc. without repeating. A well known irrational number is the decimal value for pi () : 2.141592653 etc.

Modern computers have been programmed to determine the value of pi. Despite their incredible speed and having computed billions and trillions of decimal places, they have not found the end of the sequence and the sequence does not ever repeat itself! While we can understand how to construct and compute irrational numbers, we just can not find any way to classify them in relation to other number systems.

Real Numbers: This system of numbers includes all those types of numbers discussed thus far. A Venn diagram can be drawn to represent their relationship:



Thus, real numbers include all rational and irrational numbers. The rational numbers include all integers which include all whole numbers which include all natural numbers.

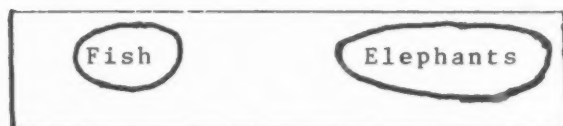
Are there "unreal" numbers? Of course. You may encounter an Imaginary Number symbolized as " i ." So far as we can imagine, no number times itself will yield a negative number. Thus, the

$\sqrt{-7}$, the $\sqrt{-310}$, etc. are all imaginary. On the diagram above they are shown as falling outside the real number system.

3.4 SET THEORY

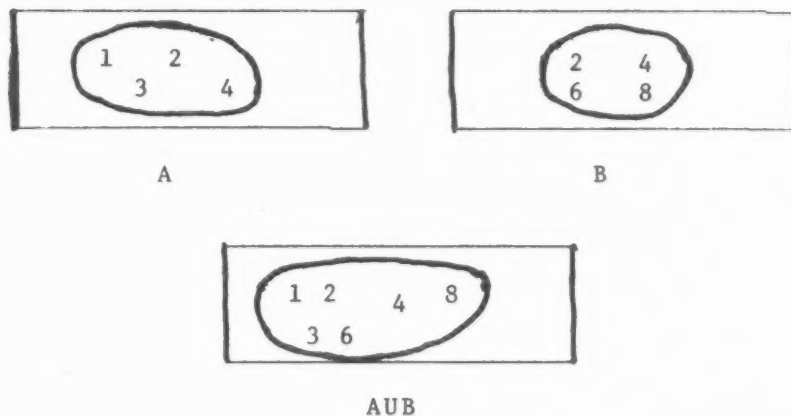
A set theory is a collection of things. For example, a set of dishes, a set of chairs, a set of people, etc. Most of the time in mathematics we talk about sets of numbers and contain the members or elements of the sets in braces. For example, the set of counting numbers less than five is (1, 2, 3, 4).

- 3.4.1 Venn Diagrams are pictorial representations of sets usually by circles contained within a large rectangle representing the universal set; that is the set of all sets. For example, if the universal set was the set of all animals then the set of all fish and the set of all elephants may be represented as follows:



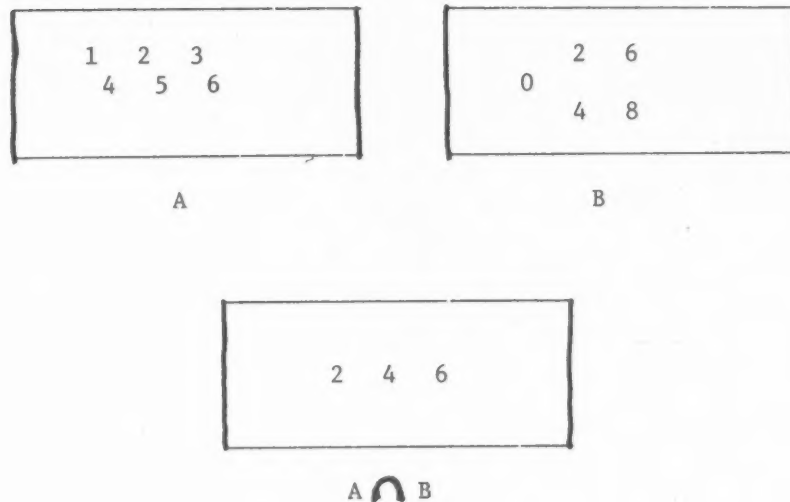
Note that the two circles do not cross since no fish are elephants.

- 3.4.2 Union of two sets means that the elements of the sets united are all put together creating a new probably larger set. For example, say Set A is (1, 2, 3, 4) and Set B is (2, 4, 6, 8) the A union B ($A \cup B$) is (1, 2, 3, 4, 6, 8). By Venn diagrams:



- 3.4.3 Intersection means those elements of sets that are common to both sets. For example, say Set P has elements (1, 2, 3, 4, 5) and Set Q has elements (0, 2, 4, 6, 8) then the intersections is those elements in both sets or $A \cap B \rightarrow (2, 4, 6)$.

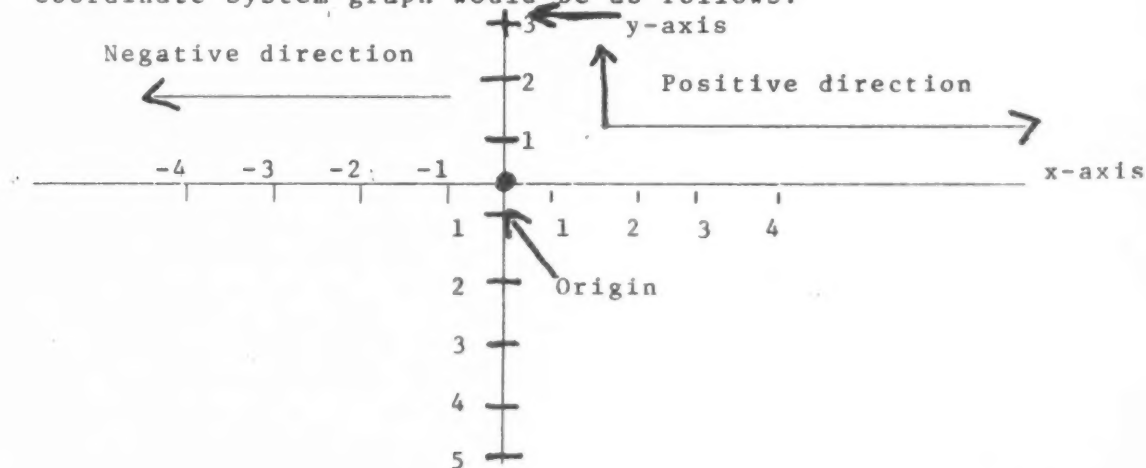
By Venn diagrams the intersection may appear as follows:



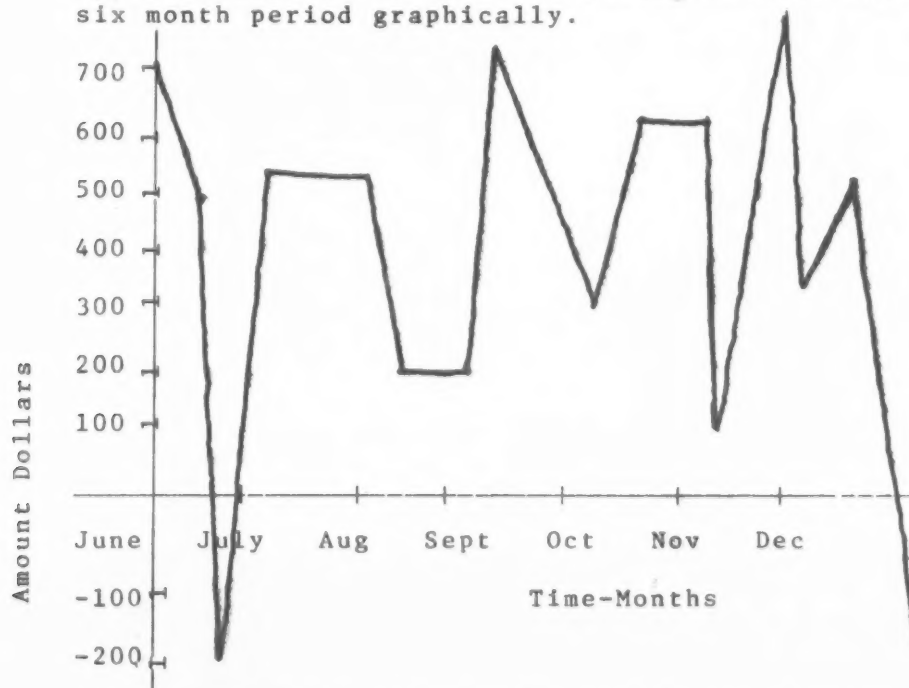
- 3.4.4 The logical use of sets can be applied to many aspects of daily living. In biology in the study of heredity sets of genes may be used to predict the structural makeup of offspring. In computer algorithms, many times the complexity of a process, the algorithm, may be simplified by Venn Diagrams.

3.5 GRAPHS

A graph may be thought of as a comparative picture showing visually how two things compare. Usually this is done by intersecting two lines at a common point called the origin and allowing values to be assigned along regular space intervals of the line. A typical display for the rectangular coordinate system graph would be as follows:

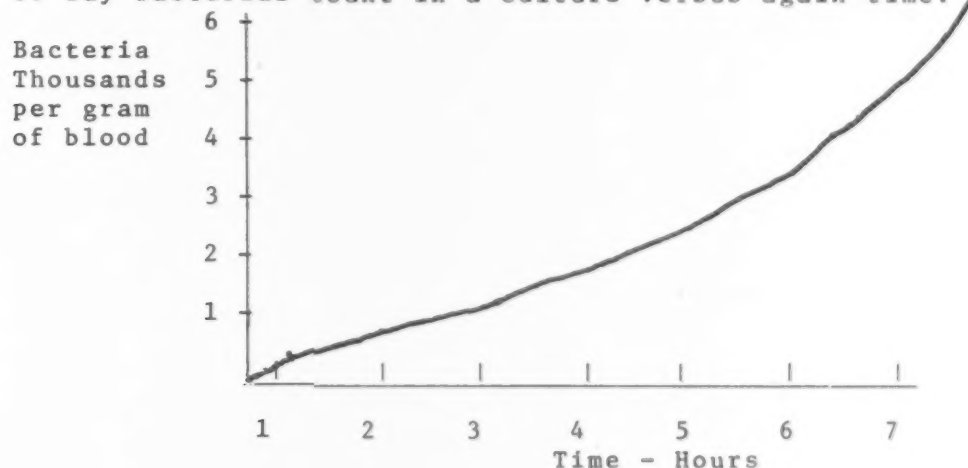


3.5.1 Many observations in our everyday existence are in relation to time. For an example, let's look at typical American Joe Doe's checking account over a six month period graphically.



A graph has much information within its display. Notice that Joe's account gets more positive the first of each month, a horizontal line means that Joe is not drawing on the account at that time and when the graph dips

- 3.5.2 The main purpose of a graph is to show the relationship of two quantities. Most all of higher mathematics deals exactly with this concept of comparison through functions. Another important concept relating to graphs is that of the steepness or slope of the curve at particular points. The slope tells us how abruptly the comparison is taking place. Let us look at an example of say bacterial count in a culture versus again time.

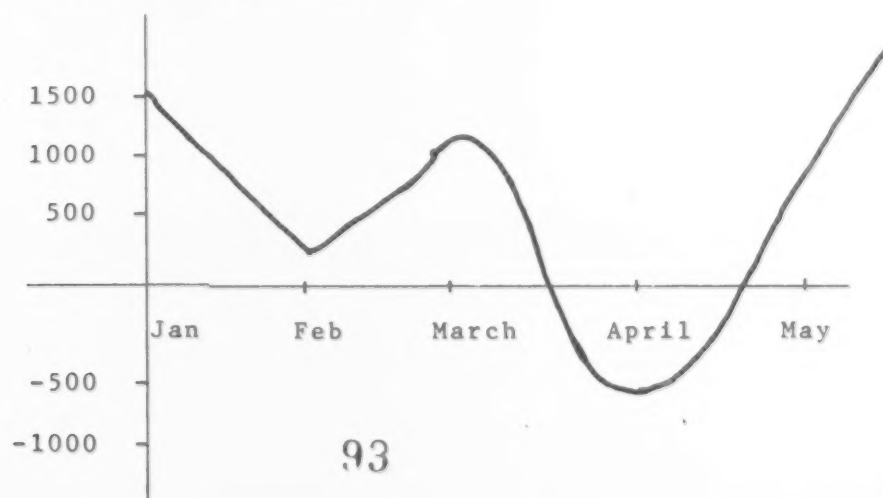


Notice how the graph gets steeper as time increases. This is an example of an exponential curve. Many things in nature behave exponentially in comparison to time such as human population, radioactive decay, etc.

3.5.3 Examples

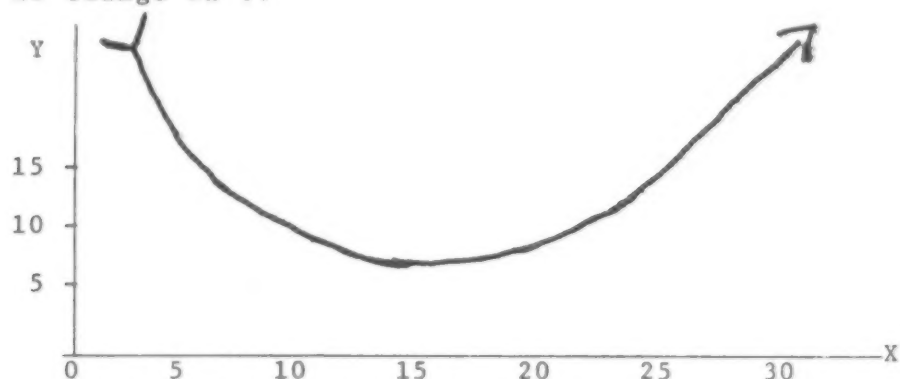
Try using the material you have learned so far in interpreting the following graphic problems.

- (1) Approximately how much money does Joe have in the account on March 15th?



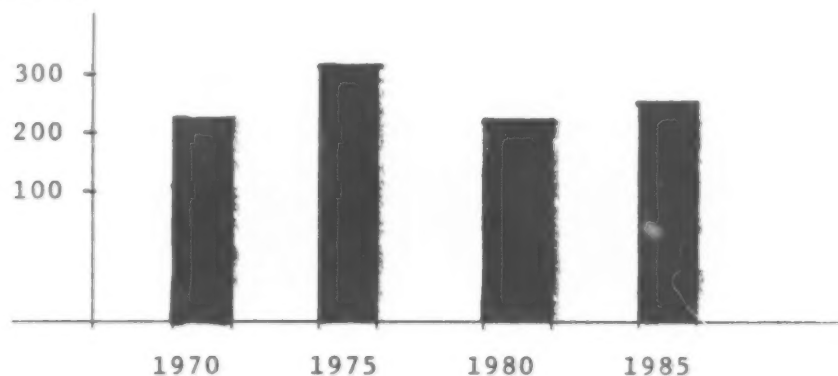
- (a) \$500.00 (b) \$0.00 (c) \$-700.00 (d) \$1,500.00
 (e) Don't know

- (2) Approximately for what value of X is there little or no change in Y.



- (a) 0 (b) 10 (c) 20 (d) 30 (e) Don't know

- (3) In what year was the population of the U. S. the greatest?



- (a) 1970 (b) 1975 (c) 1980 (d) 1985 (e) None of them.

4.1 METRICS

The metric system is a universal system of measurement based on multiples of 10 for the comparison of concepts. It is much easier than the English system.

For example, in the English system we have many cumbersome conversions:

1 foot = 12 inches
 1 yard = 36 inches
 1 mile = 5280 feet
 1 pound = 16 ounces
 1 pint = 16 ounces
 1 tablespoon = 3 teaspoons

See, there is no logical reason for units.

In the metric system there is logic and order. For example.

1 meter = 100 centimeters
 1 kilogram = 1000 grams
 $500 \text{ milliliter} = \frac{500}{1000} = 1/2 \text{ liter}$

and so on.

4.1.1 Just like in any system of measurement we have four concepts to measure; they are mass, length, volume and time. Time in the metric system uses second just like the English system. These fundamental units of measurement are named as follows:

<u>Concept</u>	<u>Unit</u>
time	second
mass	gram (about the weight of a paper clip)
length	meter (about 3 inches longer than a yard)
volume	liter (slightly more than a quart)

4.1.2 Since all the metric system is based on units of ten we give names to each multiple concept as follows:

<u>Prefix</u>	<u>Fractional meaning</u>
milli	1/1000 or .001
centi	1/100 or .01
deci	1/10 or .1
(unit)	1
deka	10
hecto	100
kilo	1000

4.1.3 To make any conversions from one prefixed unit to another in the metric system is easy. The rules are as follows:

- (A) When converting from larger to smaller units, multiply by ten on each move up until you reach the conversion you want.

For example, convert 5 decimeters to millimeters.

Look at partial prefix scale.

milli	Note that you must make two moves
centi	to go from deci to centi. Thus
deci	multiply by 10 each move.

So, 5 decimeters = $5 \times 10 \times 10$ centimeters = 500 centimeters.

Another form of the rule is to move the decimal point once to the right for each move up.

- (B) When converting from smaller units to larger units, divide by ten on each move or simply move your decimal point once to the left for each move.

For example, convert 63800 centimeters to kilometer. Again, look at our prefix scale:

milli
centi
deci
unit
deka
hecto
kilo

$$63800 \text{ cm} = \frac{63800}{10 \times 10 \times 10 \times 10 \times 10} = .638 \text{ km}$$

The [=] key erases the temporary register (memory). Pressing the clear key ([C] or [CL]) twice will also erase this register.

4.2.3 Operations

The following operations may be performed by depressing the key indicated:

ADDITION - Depress the [+] key.

SUBTRACTION - Depress the [-] key.

MULTIPLICATION - Depress the [X] key.

DIVISION - Depress the [$\div$] (divide) key.

CLEAR - Depress the [CL] (clear) key to clear an incorrect entry on the keyboard. On some calculators, this key may be depressed twice to clear the register and the memories. On others, another key is provided for this purpose. The clear key may be labeled with a C.

PERCENT - Depress the [%] (percent) key to enter numbers as percents in both multiplication and division problems.

SQUARE ROOT - Depress the [$\sqrt{}$] key after entering the number. The display will yield the square root of that number.

More complicated calculators will have many other keys. These will not be of any help unless you plan to study the manual carefully. Many students find the extra keys confusing.

4.2.4 Examples

To add numbers, enter the numbers in the same order that they are read.

Example: To add $45 + 23 =$
 Press [4], press [5], press [+], press [2],
 press [3], press [=].

To subtract numbers, enter the numbers in the same order that they are read. Enter the top number (minuend) first and the bottom number (subtrahend) second.

Example: To subtract $45 - 23 =$
 Press [4], press [5], press [-], press [2],
 press [3], press [=].

To multiply, enter the numbers in the same order that they are read. The top number (multiplicand) or the bottom number (multiplier) may be entered first; the answer will be the same.

Example: To multiply 78
x16

Press $\boxed{7}$, press $\boxed{8}$, press $\boxed{\times}$, press $\boxed{1}$,
press $\boxed{6}$, press $\boxed{=}$.

To divide, enter numbers in the same order that they are read. Always enter the top number (dividend) first and the bottom number (divisor) last.

Example: To divide 78
 $\div 16$

Press $\boxed{7}$, press $\boxed{8}$, press $\boxed{\div}$; press $\boxed{1}$,
press $\boxed{6}$, press $\boxed{=}$.

Add the following: $39 + 87 =$
 $23 + 35 =$

Subtract the following: $907 - 342 =$
 $389 - 324 =$

Multiply the following: $378 \times 14 =$
 $987 \times 59 =$

Divide the following: $144 \div 12 =$
 $327 \div 9 =$

4.2.5 Memory

On many calculators a number may be stored in a special memory key which saves that number for future use. To add a number to the special memory, press the $\boxed{M+}$ key. To subtract a number from that memory, press the $\boxed{M-}$.

In order to use the number in this special memory, you will have to press the memory recall $\boxed{MR}$ key. The number will appear on the visual display and you can now perform any operation on that number ($\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, etc.) . A useful application might involve getting a monthly sales total, for February, pressing the $\boxed{M+}$ after getting the January total; getting a total for February, pressing the $\boxed{M+}$ again; and so forth for all 12 months. When you have finished with 12 months, just press the memory recall key $\boxed{MR}$ and the yearly total will appear on the visual display. Each month has been added in memory. Then, press $\boxed{\div}$ followed by $\boxed{1}$, $\boxed{2}$, and $\boxed{=}$. Of course, this divided the annual total by 12, resulting in the monthly average (displayed). To clear this special memory, you will have to press the Clear Memory $\boxed{CM}$ key if there is one. On some other calculators, pressing the clear key ($\boxed{C}$ or $\boxed{Cl}$) twice will clear the memory. Of course, cutting off the calculator will also work.

4.3 COMPUTERS

This brief section on computers can do little more than acquaint you with the most elementary concepts concerning computers. It can serve as a good starting point for more study or as a simple survey of how they work.

4.3.1 Machine Language Concepts

All of us have seen the output of computers, but we can appreciate them a little better if we look at the simplest level of what the machine understands. All modern computers are constructed from simple "on" and "off" switches. Modern circuitry from Large Scale Integration (LSI) "chips" allow thousands and even millions of possible on-off switches. To keep track of all these switches, the designer uses only two digits: 0 = off and 1 = on. Thus, modern computers are called digital computers.

Obviously, a lot of organizing has to go into arranging these millions of 1's and 0's to do all that we know they can. Each switch contains a tiny bit of information (on or off). Arranging bits in sequences provides more information. Usually a sequence of eight bits is used:

```
10000000 - may stand for number 1
01000000 - may stand for number 2
11000000 - may stand for number 3
00100000 - may stand for number 4
```

and so on. Letters of the alphabet can also be represented with 8 bit sequences.

The central processor of the computer can usually work with one or two or more of these sequences at a given instant. Thus, an 8 bit processor can deal with a single 8 bits sequence at a time. A 16 bit processor can work with two such sequences, and a 32 bit processor with four such sequences. Microcomputers started with 8 bit processors (many today use 16 bit processors) while large, timesharing computers have a much larger processor (32 bit, 64 bit, etc.) .

The key to appreciating all of this is understanding the incredible speed with which all of this occurs. Tiny electrical currents flow through these switches at almost the speed of light (186,000 + miles per second) ! The circuits may be only as short as a fraction of an inch. In sum, that is millions of switches per second.

4.3.2 Higher Level Languages

Only the most specialized designers complete the tedious work of arranging the 8 bit sequences in Bytes (a character of the alphabet). Programming languages are arranged so that several such characters (bytes) yield a command or word which the computer will obey. Thus, a programmer may type the characters L, E, T in on a computer's keyboard. He does not even have to know about bits or bytes. He may have only learned a high level language where LET has a meaning to command the computer. His job is to arrange the commands into a program which will accomplish a specific task, play a game, draw a picture, etc.

The most popular language learned by first-time programmers is the Beginner's All Purpose Symbolic Interaction Code (BASIC). The statements in BASIC usually involve English words (LET, PRINT, GO TO, IF A THEN B, etc.). BASIC is quite powerful ("all purpose") and is the only language that most computer enthusiasts ever learn. Other languages have specialized uses:

Language Name	Which Stands For	Used For
FORTRAN	Formula Translator	Math/Science
COBOL	Common Business Oriented Language	Business

All of these languages will accomplish most computing applications. PASCAL is a relatively new all-purpose language which has great power for some applications.

4.3.3 General Systems Analysis (GSA)

Understanding the complex levels and operating capabilities of computers has produced a relatively simple classification system which provides a framework for approaching most computer concepts. Only four elements are involved:

Input - Those bits or language statements coming into the Central Processing Unit (CPU).
Input also includes memory from various parts of the computer or outside the computer (see Section 4.5.4).

Processing - Two functions of the Central Processing Unit (CPU) lead most computer enthusiasts to consider it the "brain" of the computer:

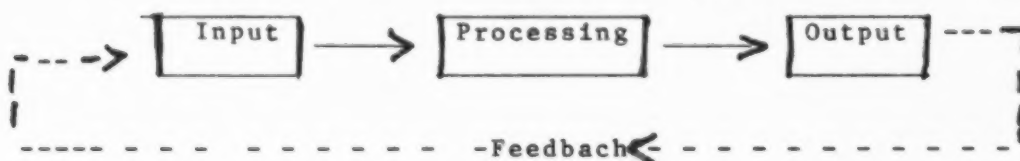
Managerial Functions: The CPU coordinates activities, calling up memory, sending electrical information back to memory, and executing the many functions required of all parts in the computer.

Arithmetic Logic Unit (ALU): The ALU has built in circuits which manipulate the bytes (e.g., 8 or 16 bit sequences) performing calculations and logical comparisons.

Output - The information coming out of the CPU can go (1) to the video screen so that users can read what the CPU has done; (2) to the printer so that users can get a written copy of what the CPU has done; or (3) back into the various memory registers for later use.

Feedback - The user or programmer can evaluate the output (much like the CPU) and send new messages as input to the CPU. These new messages may include commands to modify the operations of the program which is being carried out by the CPU. This constant modification is classified as feedback.

These four elements are often diagrammed as a feedback circuit:



The GSA framework has broad flexibility in classifying activities of a computer, a person, etc.

4.3.4 Memory

The memory in a computer seems mysterious until we just think of the switches in all of that circuitry. As long as current is flowing through the tiny switches, they maintain ("remember") their on or off position. Cut off the power, and the computer "forgets" anything that is not "wired in" or written down in permanent circuits.

Sequences of bits and bytes are stored in several ways:

- A. **External:** Disks, punched cards, tape, etc., are used to keep programs (instructions to the computer) and data. On microcomputers, we can easily see these external storage devices in the "floppy disks" or cartridges used. The point of external storage is to keep many things which can be used, one at a time, on the computer. If we attempted to keep all of this internally, the computer would soon run out of memory space. External memory is preserved, even when the computer is off.

- B. Internal: Temporary memory registers are activated as needed to store information for the CPU to work with. The most important type of memory is Random Access Memory (RAM). RAM is the memory which can be used and coordinated by the CPU at a given point in time. While the CPU of a microcomputer can process only 8 bits at a time, its managerial functions can keep thousands of bytes (and millions of bits) in a ready state for use.

4.4 PLANE GEOMETRY (TWO DIMENSIONS)

4.4.1 Polygons

The word polygons means a many-sided figure contained within a flat surface, a plane. These figures may have any number of sides and must be closed. Several shapes that every person should be aware of are as follows with their names.

3 sided - triangles

4 sided - quadrilaterals

parallelograms - opposite sided parallel
under which come: squares,
rectangles, rhombus

trapezoid - one pair of paralld sides
with still 4 sides

5 sides - pentagons

6 sides - hexagons

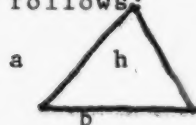
8 sides - octagons

10 sides - decagons

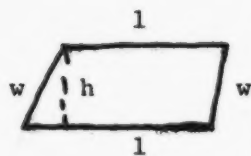
infinite sides - circles

4.4.2 Polygons - Perimeter and Area

Some important formulae for geometrical shapes are as follows:

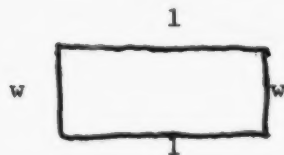


Perimeter - add up all sides $P = a+b+c$
Area - multiply one-half the base (b)
by the perpendicular height (h).
 $A = 1/2 bh$



General Quadrilateral - Perimeter is
two lengths plus two widths. $P = 2(l+w)$.

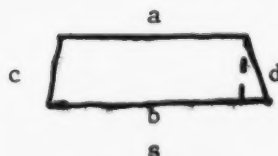
Area is l multiplied by the perendicular
height (h). $A = lxh$



Rectangle - a quadrilateral with 4 90°
angles.

Perimeter: $P = 2(l+w)$

Area: $A = lxw$



Trapezoid - Perimeter equals $P = a+b+c+d$
Area: $A = 1/2 (a+b)$



Square - a rectangle with 4 sides of equal
length.

Perimeter: $P = 4xs$

Area: $A = sxs$ or $A = s^2$

4.4.3 Circle

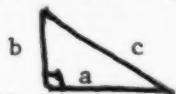
A circle may be thought of as a polygon with an infinite number of sides. However, the definition of a circle is a set of points in a plane equidistant from a given point called the center. Since the distance around a circle, called its circumference, when divided by the distance across the circle going through the center always gives the same non-ending, non-repeating decimal number of 3.14159265, etc., we give the number a name, Pi with a symbol π .

Therefore, the circumference $C = \pi D$, circumference equals Pi times the diameter. Since one-half the diameter is called a radius we may write $C = 2\pi R$.

Now the area for a circle is $A = \pi R^2$ or the area is Pi multiplied by the radius times the radius again.

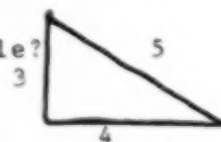
4.4.4 Pythagorean Theorem

This theorem or observation accepted as always true applies only to triangles containing a 90° angle within. The side opposite the 90° angle is called the hypotenuse; the other two are called legs.



The pythagorean theorem states that the sum of the square of the legs equals the hypotenuse squared for a right triangle. In formula form referring to the drawing this means $a^2 + b^2 = c^2$.

Example: Is the triangle a right triangle?



Since the hypotenuse is always the longest side, in this case 5, we will test the pythagorean theorem.

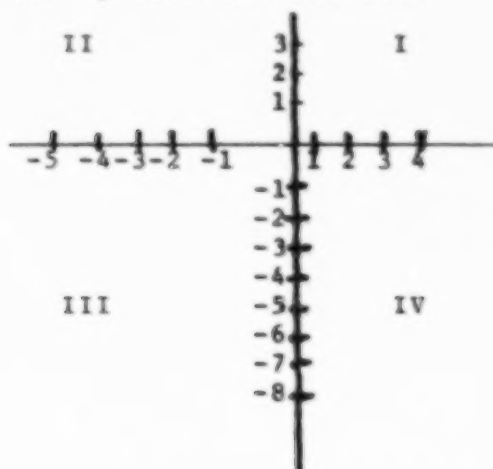
$$\begin{array}{rcl} & 2 & 2 & 2 \\ \text{Solution: } & a & + & b & = & c \\ & 3 & + & 4 & = & 5 \end{array}$$

$$\begin{array}{rcl} 9 & + & 16 & = & 25 \\ 25 & = & 25 \end{array}$$

So, yes it is a right triangle.

4.4.5 Coordinate Geometry

Coordinate geometry first is applied to a plane called plane geometry. Introduced is the Cartesian Coordinate Plane which is simply two number lines crossed at their origins. The horizontal line is called normally the x-axis or abscissa and the vertical line is called the y-axis or ordinate.



Note that by intersecting these two lines, four parts or quadrants are formed.

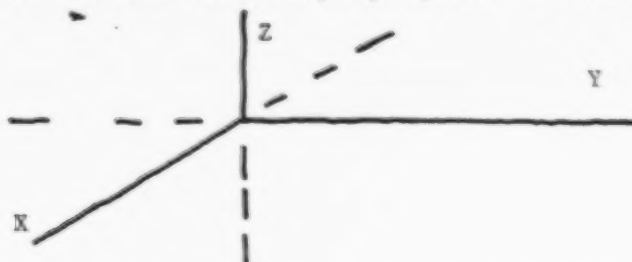
x-axis - positive is right of origin; negative is left of origin

y-axis - positive is up from origin; negative is down from origin

Any point or position may be specified by an ordered pair of numbers called coordinates. The first number specifies the X position of the point and the second specifies the Y position. For example, the point, say Q, with coordinates (3, -6) would lie in the IV quadrant three units to the right and six units down from the origin.

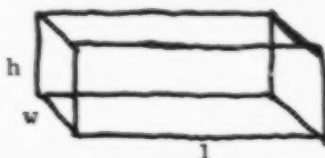
4.5 SOLID FIGURE GEOMETRY (THREE DIMENSIONS)

- 4.5.1 Coordinate geometry must be also applied to three dimensions since most of our life experience are in space rather than planes. Position must now be specified by three numbers or coordinates. These are positions on three axes - an, X, Y, and Z axis.



Think of the X and Y axis as being in the plane of the floor of a room and the Z axis being the corner at two walls. This gives the concept of three dimensions.

- 4.5.2 Important three dimensional shapes are rectangular solids, cylinders and spheres. The area of the surface comprising these figures is called surface area.



The rectangular solid is merely a box having length, width and height.

It has six surfaces or faces comprising the figure. Opposite sides are exactly alike or congruent; therefore, we have two bases, two ends and two sides. The total area for this is $SA = 2(lw + wx + lx)$.

- 4.5.3 A cylinder can be made from a rectangle and two circles. If you curl a rectangle from side and to side end then an open cylinder will be formed. The length of the preformed rectangle is equal to the circumference of the circle. Thus, the surface area equals the area of the rectangle plus the area of two circles.
 $SA = 2\pi rh + 2\pi r^2$ for a cylinder.

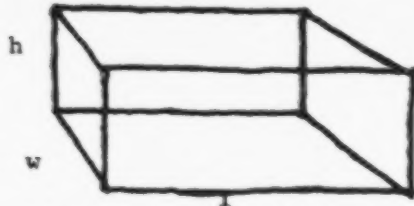


A sphere is a set of points in space equidistant from a given point called the center. An example of a sphere is a round ball.

The surface area of a sphere is $SA = 4\pi R^2$.

4.5.4 Volume may be thought of as the amount of space within a figure. Also, it may be thought of as area multiplied by a third dimension.

The volume of a rectangular solid is then the area of its base multiplied by its height or $V = l \times w \times h$.



The volume of a cylinder is then the area of its base multiplied by its height. So $V = \pi R^2 h$.



The volume of a sphere is not so intuitive. The formula for the volume of a sphere is $V = \frac{4}{3} \pi R^3$.

